Jean-Marie Dufour
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McGill University
ECON 763
Financial econometrics
Mid-term exam

Time allowed: 1.5 hour

40 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)

(a) If a random variable X has mean mean zero and variance equal to one, then $\mathbb{P}[|X| > 2] \leq 0.25$.

(b) The mean of a random variable is always smaller than its median.

(c) Any strictly stationary Gaussian process is in L_2.

(d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.

(e) Hilbert space theory is not applicable to non-stationary processes.

(f) When applied to second-order stationary processes, the projection theorem (from Hilbert space theory) implies that optimal forecasts are uncorrelated with the past of the process.

(g) The Wiener-Kolmogorov optimal forecasting formula is only applicable to finite-order moving average processes.

(h) Non-invertible moving processes have no autocorrelation generating function.

40 points 2. Consider the following models:

$$X_t = 10 + u_t + 0.5\, u_{t-1} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0, 1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;
ii. $\gamma(k)\,,\ k = 1,\, \ldots\,,\, 8;$
iii. $\rho(k)\,,\ k = 1,\, 2,\, \ldots\,,\, 8.$

(d) Graph $\rho(k)\,,\ k = 1,\, 2,\, \ldots\,,\ 8.$

(e) Find the coefficients of $u_t,\ u_{t-1},\ u_{t-2},\ u_{t-3}$ and u_{t-4} in the moving average representation of X_t.

(f) Find the autocovariance generating function of X_t.

(g) Compute the first two partial autocorrelations of X_t.

(h) If $X_{10} = 11$, compute the best linear forecast of X_{11} based on X_{10} (only). Justify your answer.

(i) If $u_{10} = 1$ and $X_{10} = 11$, can you compute the best linear forecast of X_{11} and X_{12} based on the past X_t up to time 10? If so, compute these optimal forecasts.

20 points 3. Let $\rho(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

(a) $\rho(0) = 1$ and $\rho(k) = \rho(-k)$, $\forall k \in \mathbb{Z}$;

(b) $|\rho(k)| \leq 1$, $\forall k \in \mathbb{Z}$;

(c) the function $\rho(k)$ is positive semi-definite.