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McGill University
ECON 763
Financial econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

10 points 1. Consider a process that follows the following model:

$$X_t = \sum_{j=1}^{m} [A_j \cos(\nu_j t) + B_j \sin(\nu_j t)] \, , \; t \in \mathbb{Z} \, ,$$

where $\nu_1, \ldots, \nu_m$ are distinct constants on the interval $[0, \, 2\pi)$ and $A_j, \, B_j, \, j = 1, \, \ldots \,,$ m, are random variables in L_2 , such that

$$\begin{aligned} E(A_j) &= E(B_j) = 0 \, , \, E(A_j^2) = E(B_j^2) = \sigma_j^2 \, , \; j = 1, \, \ldots \, , \, n \, , \\ E(A_j A_k) &= E(B_j B_k) = 0, \text{ for } j \neq k \, , \\ E(A_j B_k) &= 0, \, \forall j, \, k \, . \end{aligned}$$

(a) Show that this process is second-order stationary.

(b) For the case where $m = 1$, show that this process is deterministic.

40 points 2. Consider the following ARMA model:

$$X_t = 0.5 \, X_{t-1} + u_t - 0.25 \, u_{t-1} \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0,1)$ sequence. Answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

i. $E(X_t)$;

ii. $\gamma(k)$, $k = 1, \ldots, 8$;

iii. $\rho(k)$, $k = 1, 2, \ldots, 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots, 8$.

(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Compute the first two partial autocorrelations of X_t.

(g) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{10}, X_{11}, X_{12} and X_{13} based on X_{10} (only)? If so, compute these.

(h) If $X_{10} = 1$, $u_{10} = 2$, $u_9 = 1$, $u_8 = 0.99$, $u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{11}, X_{12} and X_{13} based on the history of the process up to X_{10}? If so, compute these.

15 points 3. Let $X_1, X_2, \ldots, X_T$ be a time series where $X_1, X_2, \ldots, X_T$ have continuous distributions.

(a) Propose a method for testing the hypothesis that $X_1, X_2, \ldots, X_T$ are independent and identically distributed (i.i.d.) without any assumption on the existence of the moments for $X_1, X_2, \ldots, X_T$.

(b) If $X_1, X_2, \ldots, X_T$ have common median m_0, describe a procedure for testing whether these observations are independent without assuming identical distributions.

(c) Consider the "median regression" model:

$$y_t = x_t'\beta + u_t,\ t = 1, \ldots, T, \tag{2}$$

where x_t, $t = 1, \ldots, T$, are $k \times 1$ fixed vectors and the the disturbances u_t, $t = 1, \ldots, T$, are independent with median zero and continuous distributions. Propose procedures for testing hypotheses of the form $H_0 : \beta = \beta_0$ and build confidence sets for β.

20 points 4. Let R_{it}, $i = 1, \ldots, n$, be returns on n securities for period t, and $\tilde{R}_{Mt}$ the return on a benchmark portfolio $(t = 1, \ldots, T)$. The (unconditional) CAPM which assumes time-invariant *betas* can be assessed by testing:

$$\mathscr{H}_{\mathrm{E}} : a_i = 0\,, \quad i = 1, \ldots, n, \tag{3}$$

in the context of the MLR model

$$r_{it} = a_i + \beta_i \tilde{r}_{\mathrm{M}t} + \varepsilon_{it}\,, \quad t = 1, \ldots, T,\ i = 1, \ldots, n, \tag{4}$$

where $r_{it} = R_{it} - R_{ft}$, $\tilde{r}_{\mathrm{M}t} = \tilde{R}_{\mathrm{M}t} - R_{ft}$, R_{ft} is the riskless rate of return and ε_{it} is a random disturbance, such that

$$V_t \equiv (\varepsilon_{1t}, \ldots, \varepsilon_{nt})' = JW_t \;,\; t = 1, \ldots \;,\; T \,, \tag{5}$$

where J is an unknown, non-singular matrix and the distribution of the vector $w = vec\,(W)$, $W = [W_1, \ldots \,, W_T]'$ is either: (i) known (hence, free of nuisance parameters), or (ii) specified up to an unknown finite dimensional nuisance-parameter (denoted ν).

(a) Put the model (4) in matrix notation.

(b) On assuming that the vectors $W_1, \ldots \,, W_T$ are i.i.d. $N[0, I_n]$, describe the likelihood ratio test for $\mathscr{H}_E$, and discuss how this test could be implemented.

(c) Propose a procedure for testing whether the errors $W_1, \ldots \,, W_T$ are i.i.d. $N[0, I_n]$.

(d) If another distribution is assumed for w (such as a heavy-tailed distribution), discuss how such a test could be implemented.

15 points 5. Consider a time series of asset returns R_t, $t = 1, \ldots, T$, which are i.i.d. according to stable distribution, with characteristic function

$$\ln \int_{-\infty}^{\infty} e^{ist} d\mathrm{P}(S < s) = \begin{cases} -\sigma^{\alpha}|t|^{\alpha}[1 - i\beta \text{ sign}(t) \tan\frac{\pi\alpha}{2}] + i\mu t, & \text{for } \alpha \neq 1, \\ -\sigma|t|[1 + i\beta\frac{\pi}{2} \text{ sign}(t) \ln|t|] + i\mu t, & \text{for } \alpha = 1. \end{cases} \tag{6}$$

(a) Discuss the interpretation of the different parameters μ, σ, α and β.

(b) Why are stable random variables called “stable”?

(c) On assuming that $\beta = 0$, propose a method for testing

$$H_0(\alpha_0) : \alpha = \alpha_0. \tag{7}$$

(d) On assuming that $\beta = 0$, discuss how a confidence set for α could be built.