

McGill University
ECN 706
Special topics in econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

- 20 points 1. Define the following notions:
- (a) sufficient statistic;
 - (b) ancillary statistic;
 - (c) Fisher information;
 - (d) complete statistic.

- 30 points 2. Consider the following equilibrium model:

$$\begin{aligned}Q_t &= a + bp_t + u_{1t}, \\p_t &= c + dQ_{t-1} + u_{2t}, \\Q_0 &\text{ is fixed}\end{aligned}$$

where the disturbances $(u_{1t}, u_{2t})'$, $t = 1, \dots, T$ are independent $N[0, I_2]$, Q_t represents the quantity sold, and p_t the price. For which parameters is the vector $p = (p_1, \dots, p_T)'$

- (a) exogenous for (a, b) ?
- (b) exogenous for (c, d) ?
- (c) sequentially exogenous for (a, b) ?
- (d) sequentially exogenous for (c, d) ?
- (e) strongly exogenous for (a, b) ?
- (f) strongly exogenous for (c, d) ?

Justify your answers.

- 20 points 3. Demonstrate the following relationship between identifiability and unbiased estimation:
if a function $g(\theta)$ of a parameter θ is not identifiable, then there is no unbiased estimator of $g(\theta)$.

- 30 points 4. Consider the linear regression model

$$y = X\beta + u$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \dots, \beta_k)'$, and $u = (u_1, \dots, u_T)'$ is a $T \times 1$ vector of unobserved error terms. Suppose the elements of u are independent and identically distributed according to a $\sigma t(1)$ distribution, where $t(1)$ represents a Student t distribution with 1 degree of freedom and σ is an unknown constant.

- (a) Propose a method for testing the hypothesis $H_0 : \beta_1 = 1$ at level $\alpha = 0.05$ in the context of this model such the size of the test is exactly equal to $\alpha = 0.05$.
- (b) Propose a test for detecting serial dependence between the errors u_1, \dots, u_T such the size of the test is exactly equal to $\alpha = 0.05$.