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McGill University
ECN 706
Special topics in econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

30 points 1. Provide brief answers to the following questions (maximum of 1 page per question).

(a) Explain the difference between the "level" of a test and its "size".

(b) Explain the difference between the "level" of a confidence set and its "size".

(c) Discuss the link between tests and confidence sets:

i. How can a confidence set be derived from a family of tests ?

ii. How can a test be derived from a confidence set ?

iii. If a confidence set for a parameter θ is derived from a family of tests with level α, what is the level of this confidence set ? Justify your answer.

20 points 2. Let $\ell(Y;\theta)$ be the likelihood function for the sample $Y = (Y_1, \ldots, Y_n)'$. Show that

$$I(\theta) = E\left[-\frac{\partial^2 \log \ell(Y;\theta)}{\partial\theta\,\partial\theta'}\right].$$

20 points 3. Demonstrate the following relationship between identifiability and unbiased estimation:
if a function $g(\theta)$ of a parameter θ is not identifiable, then there is no unbiased estimator of $g(\theta)$.

30 points 4. Consider the linear regression model

$$y = X\beta + u \tag{0.1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ fixed matrix of explanatory variables (observed), $\beta = (\beta_1, \ldots, \beta_k)'$, and $u = (u_1, \ldots, u_T)'$

is a $T \times 1$ vector of unobserved error terms. Suppose the elements of u are independent and identically distributed according to a $\sigma t(1)$ distribution, where $t(1)$ represents a Student t distribution with 1 degree of freedom and σ is an unknown constant.

(a) Propose a method for testing the hypothesis $H_0 : \beta_1 = 1$ at level $\alpha = 0.05$ in the context of this model such the size of the test is exactly equal to $\alpha = 0.05$.

(b) Propose a test for detecting serial dependence between the errors $u_1, \ldots, u_T$ such the size of the test is exactly equal to $\alpha = 0.05$.