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McGill University
ECN 706
Special topics in econometrics
Final exam

No documentation allowed
Time allowed: 3 hours

10 points 1. Let $\ell\,(Y;\theta)$ be the likelihood function for the sample $Y=(Y_1,\ldots,Y_n)'$. Show that

$$I\,(\theta)=E\left[-\frac{\partial^2\log\ell\,(Y;\theta)}{\partial\theta\,\partial\theta'}\right].$$

20 points 2. Consider the following assumptions:

H1: the variables $Y_1,\ldots,Y_n$ are independent and follow the same distribution with density $f\,(y;\,\theta)\,,\ \theta\in\Theta\subseteq\mathbb{R}^p$;

H2: the interior of Θ is non-empty, and θ_0 belongs to the interior of Θ;

H3: the true unknown value θ_0 is identifiable;

H4: the log-likelihood

$$L_n\,(y;\,\theta)=\sum_{i=1}^{n}\log\left[f\,(y_i;\,\theta)\right]\ \text{is continuous in }\theta\,;$$

H5: $\mathsf{E}_{\theta_0}[\log f\,(Y_i;\,\theta)]$ is finite;

H6: the log-likelihood is such that $\frac{1}{n}L_n\,(y;\,\theta)$ converges almost surely to $\mathsf{E}_{\theta_0}[\log(Y_i;\,\theta)]$ uniformly in $\theta\in\Theta$;

H7: the log-likelihood is twice continuously differentiable in open neighborhood of θ_0;

H8: $I_1\,(\theta_0)=\mathsf{E}_{\theta_0}\left[-\frac{\partial^2\log f(Y;\,\theta)}{\partial\theta\,\partial\theta'}\right]$ is finite and invertible.

If $\hat{\theta}_n$ is consistent sequence of local maxima, show that the asymptotic distribution of $\sqrt{n}\left(\hat{\theta}_n - \theta_0\right)$ is $N\left[0,\ I_1\left(\theta_0\right)^{-1}\right]$.

20 points 3. State and prove the *Neyman-Pearson theorem*.

10 points 4. Define the following notions:

(a) unbiased test;

(b) α-similar test;

(c) test with Neyman α-structure.

10 points 5. Demonstrate the following relationship between identifiability and unbiased estimation:
if a function $g(\theta)$ of a parameter θ is not identifiable, then there is no unbiased estimator of $g(\theta)$.

30 points 6. Consider the standard simultaneous equations model:

$$y = Y\beta + X_1\gamma + u\,, \quad (1)$$
$$Y = X_1\Pi_1 + X_2\Pi_2 + V\,, \quad (2)$$

where y and Y are $T\times 1$ and $T\times G$ matrices of endogenous variables, X_1 and X_2 are $T\times k_1$ and $T\times k_2$ matrices of exogenous variables, β and γ are $G\times 1$ and $k_1\times 1$ vectors of unknown coefficients, Π_1 and Π_2 are $k_1\times G$ and $k_2\times G$ matrices of unknown coefficients, $u = (u_1,\ldots,u_T)'$ is a $T\times 1$ vector of structural disturbances, and $V = [V_1,\ldots,V_T]'$ is a $T\times G$ matrix of reduced-form disturbances,

$$X = [X_1, X_2] \text{ is a full-column rank } T\times k \text{ matrix} \quad (3)$$

where $k = k_1 + k_2$. and

$$u \text{ and } X \text{ are independent;} \quad (4)$$
$$u \sim N\left[0, \sigma_u^2 I_T\right]. \quad (5)$$

(a) When is the parameter β identified? Explain your answer.

(b) When is the parameter β weakly identified? Explain your answer.

(c) Suppose we wish to test the hypothesis

$$H_0(\beta_0) : \beta = \beta_0\,. \quad (6)$$

i. Describe the standard Wald-type test for $H_0(\beta_0)$ based on two-stage-least-least squares, and describe its properties.

ii. Describe an identification-robust procedure for testing $H_0(\beta_0)$.

iii. Discuss the properties of the latter procedure if the model for Y is in fact

$$Y = X_1 \Pi_1 + X_2 \Pi_2 + X_3 \Pi_3 + V \tag{7}$$

where X_3 is a $T \times k_3$ matrix of fixed explanatory variables.