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McGill University
ECON 763
Financial econometrics
Mid-term exam

No documentation allowed
Time allowed: 1.5 hour

20 points 1. Answer by TRUE, FALSE or UNCERTAIN to each one of the following statements. Justify briefly your answer. (Maximum: one page per question.)

(a) If a random variable has finite second moments, it has finite moments at all higher orders.

(b) Any stationary process of order 5 is also stationary of order 2.

(c) Any strictly stationary process is in L_2.

(d) The Wold theorem holds for finite-order moving average processes but not autoregressive processes.

(e) Non-invertible moving processes have no covariance generating function.

20 points 2. Let $\gamma(k)$ the autocovariance function of second-order stationary process on the integers. Prove that:

(a) $\gamma(0) = Var(X_t)$ et $\gamma(k) = \gamma(-k)$, $\forall k \in \mathbb{Z}$;

(b) $|\gamma(k)| \leq \gamma(0)$, $\forall k \in \mathbb{Z}$;

(c) the function $\gamma(k)$ is positive semi-definite.

60 points 3. Consider the following models:

$$X_t = 10 + u_t - 0.75\, u_{t-1} + 0.125\, u_{t-2}, \tag{1}$$

where $\{u_t : t \in \mathbb{Z}\}$ is an $i.i.d.$ $N(0, 1)$ sequence. For each one of these models, answer the following questions.

(a) Is this model stationary? Why?

(b) Is this model invertible? Why?

(c) Compute:

   i. $E(X_t)$;
   ii. $\gamma(k)$, $k = 1, \ldots , 8$;
   iii. $\rho(k)$, $k = 1, 2, \ldots , 8$.

(d) Graph $\rho(k)$, $k = 1, 2, \ldots , 8$.

(e) Find the coefficients of u_t, u_{t-1}, u_{t-2}, u_{t-3} and u_{t-4} in the moving average representation of X_t.

(f) Find the autocovariance generating function of X_t.

(g) Find and graph the spectral density of X_t.

(h) Compute the first two partial autocorrelations of X_t.

(i) If $X_{10} = 1$ and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{10}, X_{11}, X_{12} and X_{13} based on X_{10} (only)? If so, compute these.

(j) If $X_{10} = 1$, $u_{10} = 2$, $u_9 = 1$, $u_8 = 0.99$, $u_7 = 1.2$, and assuming the parameters of the model are known, can you compute the best linear forecasts of X_{11}, X_{12} and X_{13} based on the history of the process up to X_{10}? If so, compute these.