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**ADVANCED ECONOMETRIC THEORY**  
**EXERCISES 7**  
**UNBIASED AND INVARIANT TESTS**

Reference: Gouriéroux and Monfort (1995, Chapter 15)

1. Define the following notions:
  - (a) unbiased test;
  - (b)  $\alpha$ -similar test;
  - (c) test with Neyman  $\alpha$ -structure.
2. Prove that a uniformly most powerful test with level  $\alpha$  is necessarily unbiased.
3. Let  $(Y, (P_\theta : \theta \in \Theta))$  be a parametric model. If  $\varphi(y)$  is a test of the hypothesis  $H_0 : \theta \in \Theta_0$ , where  $\Theta_0 \subseteq \Theta$ , and if  $E_\theta \varphi(y)$  is a continuous function of  $\theta$ , show the following property: if  $\varphi$  is an unbiased test with level  $\alpha$ , the test  $\varphi$  is  $\alpha$ -similar on the frontier of  $\Theta_0$ .
4. Explain how invariant tests can reduce the number of nuisance parameters in a test problem.

## References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.