

ADVANCED ECONOMETRIC THEORY
EXERCISES 14
PREDICTION AND RESIDUALS

1. Exercise 11.1 in Gouriéroux and Monfort (1995, chap. 11).
2. Exercise 11.7 in Gouriéroux and Monfort (1995, chap. 11).
3. Let

$$Y_i = x_i' \beta + u_i, \quad i = 1, \dots, n+1$$

where all the hypotheses of the classical linear model are satisfied. If $\hat{\beta}_n$ is the ordinary least squares estimator of β based on Y_1, \dots, Y_n , show that

$$\hat{Y}_{n+1} = x_{n+1}' \hat{\beta}_n \text{ and } \hat{e}_{n+1} = Y_{n+1} - \hat{Y}_{n+1}$$

are correlated.

4. Explain the difference between:
 - (a) generalized residuals,
 - (b) simulated residuals;
 - (c) two-stage simulated residuals.

5. Consider the nonlinear regression model:

$$\begin{aligned} Y_t &= f(x_t; \theta) + u_t, \quad t = 1, \dots, T+h, \quad h \geq 1 \\ u_t &= \rho u_{t-1} + \varepsilon_t, \quad |\rho| < 1 \end{aligned}$$

where x_t is fixed and ε_t is a random disturbance independent of u_{t-1}, u_{t-2}, \dots , such that $E(\varepsilon_t) = 0$.

- (a) Assuming that θ and ρ are known, compute the prediction of Y_{t+h} (in the mean-square error sense) based on Y_1, \dots, Y_T .
- (b) If θ and ρ are unknown, how would you predict Y_{t+h} ?

6. Consider a PROBIT model with the latent variable

$$Y_i^* = x_i' \theta + u_i, \quad i = 1, \dots, n,$$

where x_1, \dots, x_n are fixed and u_1, \dots, u_n are independent $N(0, 1)$ random variables. Find the generalized residuals for this model.

References

GOURIÉROUX, C., AND A. MONFORT (1995): *Statistics and Econometric Models, Volumes One and Two*. Cambridge University Press, Cambridge, U.K., Translated by Quang Vuong.