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ECONOMETRICS
REVIEW QUESTIONS
Generalized least squares

1. Consider the linear regression model

$$y = X\beta + u \tag{0.1}$$

where y is a $T \times 1$ vector of observations on a dependent variable, X is a $T \times k$ nonstochastic matrix of rank k, and u is a $T \times 1$ vector of disturbances (errors) such that

$$\begin{aligned} \mathsf{E}(u) &= 0\,, && (0.2)\\ \mathsf{V}(u) &= \sigma^2 V\,, && (0.3) \end{aligned}$$

and V is a known $T \times T$ positive definite matrix.

   (a) Is the least squares estimator of β unbiased for this model? Justify your answer.
   (b) Derive the best linear unbiased estimator of β for this model. How is this estimator called?
   (c) Define the "weighted least squares" estimator for this model and explain why this terminology is being used.
   (d) If u follows a Gaussian distribution, what is the distribution of the "weighted least squares"?

2. Explain what is meant by heterokedastic disturbances in a linear regression and how a best linear unbiased estimator could be computed in such a case.

3. Consider the model

$$y_t = x_t'\beta + u_t\,, \quad t = 1, \dots, T \tag{0.4}$$

where

$$\begin{aligned} &u_t = \rho u_{t-1} + \varepsilon_t\,, \quad t = \,\dots,\, 0,\, 1,\, 2,\, \dots && (0.5)\\ &|\rho| < 1\,, && (0.6) \end{aligned}$$

$$\{\varepsilon_t\}_{t=1}^{T} \text{ is a sequence of i.i.d. disturbances,} \tag{0.7}$$
$$\mathsf{E}(\varepsilon_t) = 0\ , \quad \mathsf{V}(\varepsilon_t) = \sigma^2\ , \forall t\ . \tag{0.8}$$

(a) Explain how the above linear regression could be transformed to make the disturbances i.i.d. (when ρ is unknown).

(b) Discuss how ρ could be estimated in the above model.

(c) Discuss how β cold be estimated in the above model.