Economics Letters 6 (1980) 43–48
North-Holland Publishing Company

THE COCHRANE–ORCUTT PROCEDURE NUMERICAL EXAMPLES OF MULTIPLE ADMISSIBLE MINIMA *

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Received 5 December 1980

We present two numerical examples of multiple admissible minima obtained by using the Cochrane–Orcutt iterative technique: a forgotten and somewhat farfetched example, constructed by Hildreth and Lu, and a new example, based on more typical economic data.

1. Introduction

Consider the usual multiple regression model with autocorrelated residuals

$$y_t = \beta_1 + \sum_{k=2}^{K} \beta_k X_{t,k} + u_t, \quad t = 1,\ldots,N, \tag{1}$$

$$u_t = \rho u_{t-1} + e_t, \quad |\rho| < 1, \quad t = \ldots,0,1,\ldots,N, \tag{2}$$

where y_t is the tth observation of the dependent variable, $X_{t,k}$ is the tth observation of the kth *exogenous* regressor ($1 \leqslant k \leqslant K$), u_t is the tth value of the disturbance term and $\beta = (\beta_1,\ldots,\beta_K)$ and ρ are parameters. Assume that the e_t are serially independent, have mean zero and constant variance σ^2.

The most widely-used approach to estimating model parameters prob-

* The Research and Development Centre of Transport Canada and the F.C.A.C. program of the *Ministère de l'éducation du Québec* have supported the research on which this article is based. The authors thank Edwin Kuh, François Vaillancourt and Daniel Racette for their help.

ably consists of combining (1) and (2) and of considering the transformed model

$$(y_t - \rho y_{t-1}) = \beta_1(1-\rho) + \sum_{k=2}^{K} \beta_k(X_{t,k} - \rho X_{t-1,k}) + e_t, \quad t=2,\dots,N, \tag{3}$$

with y_1 taken as fixed; from an initial value of ρ, the Cochrane–Orcutt (1949) iterative technique then alternately minimizes the sum of squared errors defined by (3) with respect to β conditionally on ρ and then with respect to ρ conditionally on β until successive estimates differ by arbitrarily small amounts. In a classic paper, Sargan (1964) has shown that this technique will always converge to a stationary value of the sum of squared errors [see also Oberhofer and Kmenta (1974)] and has pointed out the possibility of the existence of several local minima within the 'admissible' region where $|\rho| < 1$. The existence of several minima would mean that the Cochrane–Orcutt procedure might not converge to the global minimum because of its arbitrary starting point.

The literature is remarkably ambiguous concerning the possibility of the existence of several minima. No mention of it is made in many standard textbooks which present the Cochrane–Orcutt procedure [e.g., Goldfeld and Quandt (1972), Malinvaud (1969) and Theil (1971)]. Sargan, on the other hand, suggests investigating such a possibility by performing a fine grid search but reports that no case of multiple minima has been found in 53 cases examined by his students. Among other authors who raise the possibility of multiple minima, it is not untypical [e.g., Johnston (1972)] to mention that the use of a grid search might be advisable and then to restate the students' finding. This of course suggests that the problem is negligible with typical economic data: one is in fact left with the impression that the sum of squared residuals from the transformed model (3) may well be generally unimodal in the relevant region. To the extent of our knowledge, none of the above authors and, for that purpose, no existing textbook provides a numerical example of multiple minima; furthermore, none of them mentions that the paper by Hildreth and Lu (1960) contains such an example presented in a three-page appendix clearly entitled 'Example of dual minima'. Even Betancourt and Kelejian (1980) ignore it in a recent paper where they show (from a theoretical argument) that, in the presence of a lagged endogeneous variable among the regressors, the Cochrane–Orcutt procedure can have more than one fixed point—not altogether surprising a result if multiple minima can occur in simpler models containing only exogenous regressors. [1]

It is our intention to comment on that little-known appendix of a well-known paper and to present another numerical example of multiple admissible minima using data more typical of economic time series than those used by Hildreth and Lu.

2. Multiple admissible minima: Old and new examples

The Hildreth and Lu example may have been forgotten because of the rather unusual nature of their data. They use *five* observations, have a *single* regressor (apart from the constant term) and *non-trended*, non-serially correlated data series of which 30% of the observations are zero. By contrast, as one may verify in table 1, our data set contains *twenty-one* strictly positive observations, *four* regressors (not counting the constant) and typically *trended*, or at least serially correlated data. Our original data come from an electricity pricing study of no interest for present purposes and were modified in *ad hoc* fashion to obtain very clear results. Note that the set of explanatory variables does not contain the lagged endogenous variable.

Relevant results obtained from these data are summarized in fig. 1. The two minima of the standard error of the regression defined as $(\Sigma_{t=2}^{N} \hat{e}_t^2/n-K)^{1/2}$ where $N=21$, $n=20$ and $K=5$] were at $\hat{\rho}=0.3289$ and $\hat{\rho}=0.9318$ with a local maximum at $\hat{\rho}=0.7146$. We performed two multicollinearity tests in the neighborhood of these three values. For given values of ρ, we regressed in turn each of the transformed explanatory variables $X_{t,k}^* = X_{t,k} - \rho X_{t-1,k}$, except the constant, on the others and the constant and verified that there was no case of perfect or almost perfect fit; we then computed $(X^{*\prime}X^*)^{-1}(X^{*\prime}X^*)$, where X^* is the matrix of transformed regressors, and verified that the result was the identity matrix. These exercises were performed successfully for 10 values of ρ spaced equally within each of six distinct ranges. [2] All computations were made in single precision on a CDC Cyber 173 computer.

[1] Note also that a 'fixed point' (or 'stationary value') is not necessarily a minimum, since it can be a saddle point, although a minimum must be a fixed point. Thus, strictly speaking, Betancourt and Kelejian have not proved that multiple minima are possible in the case they have considered. Furthermore, one should remember that convergence to a saddle point is an event which occurs with probability zero [see Sargan (1964, pp. 50–51)].

[2] The ranges were: $0.28 \leqslant \rho \leqslant 0.37$, $0.67 \leqslant \rho \leqslant 0.76$ and $0.89 \leqslant \rho \leqslant 0.98$ by steps of 0.01; $0.3285 \leqslant \rho \leqslant 0.3294$, $0.7142 \leqslant \rho \leqslant 0.7151$ and $0.9314 \leqslant \rho \leqslant 0.9323$ by steps of 0.0001.

Table 1

Hildreth and Lu data

t	y	X_2
1	994	0
2	0	2
3	0	−1
4	−1000	−1
5	1000	−4

Modified electricity pricing model data

t	y	X_2	X_3	X_4	X_5
1	41 257,19	31 029,69	5,28	6,81	63,77
2	44 865,40	32 583,78	6,06	6,87	85,57
3	22 923,50	36 650,81	8,87	7,32	88,46
4	32 965,03	36 891,94	9,47	7,08	386,20
5	54 792,10	42 317,09	9,22	7,30	290,85
6	33 108,06	40 589,08	9,83	7,36	393,46
7	39 056,50	41 118,95	10,22	7,34	765,05
8	172 234,73	40 704,33	9,92	8,36	442,37
9	156 048,66	47 811,32	10,16	8,47	913,72
10	119 613,33	46 579,47	9,10	7,28	2 750,65
11	101 615,00	52 399,32	9,20	7,51	730,62
12	140 263,66	52 394,25	9,22	7,51	5 598,61
13	157 970,33	53 433,06	8,92	8,12	11 172,30
14	153 197,00	57 034,16	10,10	8,10	11 231,01
15	220 405,33	63 375,77	13,04	9,10	1 998,53
16	221 200,00	62 331,26	12,92	11,14	1 840,21
17	149 608,33	65 005,41	13,83	11,51	1 527,44
18	230 434,33	65 754,51	13,63	11,34	1 829,89
19	250 505,66	69 541,05	14,04	10,83	2 691,17
20	294 424,66	60 494,35	13,24	10,22	3 225,85
21	281 281,66	63 381,69	12,67	10,36	3 980,40

3. Conclusion

The more realistic nature of the data used here provides sufficient ground for a recommendation to combine a search routine (like the Hildreth–Lu procedure) with the Cochrane–Orcutt procedure not only, as Betancourt and Kelejian (1980) have suggested, when a lagged depen-

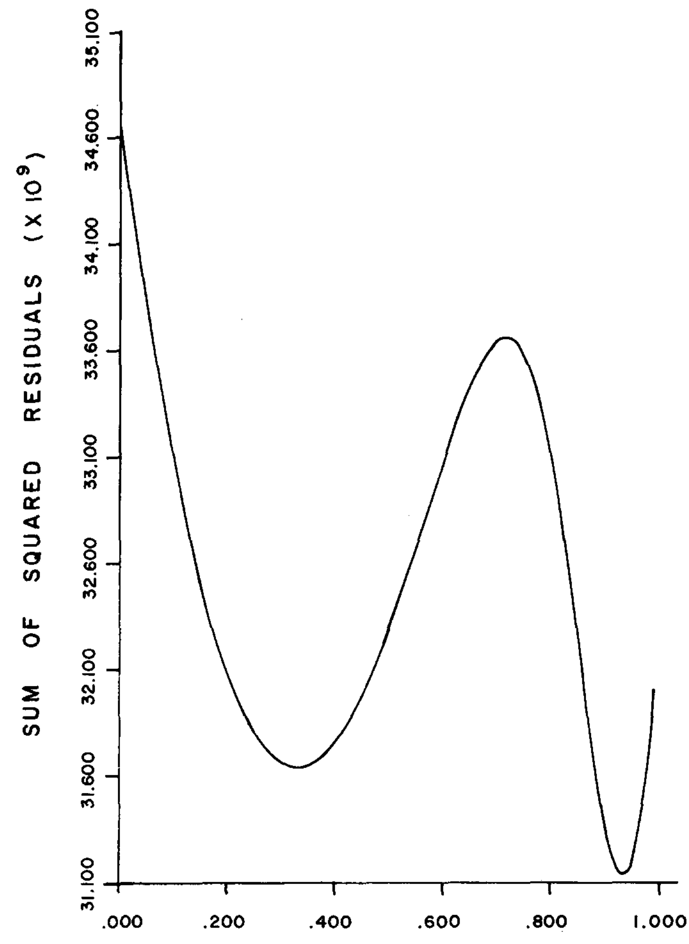

Fig. 1.

dent variables is included among the regressors but also when the list of regressors consists only of exogenous variables.

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