Measuring high-frequency causality between returns, realized volatility and implied volatility [*]

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ABSTRACT

In this paper, we provide evidence on two alternative mechanisms of interaction between returns and volatilities: the *leverage effect* and the *volatility feedback effect*. We stress the importance of distinguishing between *realized volatility* and *implied volatility* in this context, and find that *implied volatilities* are essential for assessing the *volatility feedback effect*. The leverage hypothesis asserts that return shocks lead to changes in conditional volatility, while the volatility feedback effect theory assumes that return shocks can be caused by changes in conditional volatility through a time-varying risk premium. On observing that a central difference between these alternative explanations lies in the direction of causality, we consider vector autoregressive models of returns and realized volatility and we measure these effects along with the time lags involved through short-run and long-run causality measures proposed in Dufour and Taamouti (2009), as opposed to simple correlations. We analyze 5-minute observations on S&P 500 Index futures contracts, the associated *realized volatilities* (before and after filtering jumps through the bispectrum) and *implied volatilities*. Using only returns and realized volatility, we find a strong dynamic leverage effect for the first three days. The volatility feedback effect appears to be negligible at all horizons. By contrast, when implied volatility is considered, a volatility feedback becomes apparent, whereas the leverage effect is almost the same. These results can be explained by the fact that volatility feedback effect works through implied volatility which contains important information on future volatility, through its nonlinear relation with option prices which are themselves forward-looking. In addition, we study the dynamic impact of news on returns and volatility. First, to detect possible dynamic asymmetry, we separate good from bad return news and find a much stronger impact of bad return news (as opposed to good return news) on volatility. Second, we introduce a concept of news based on the difference between implied and realized volatilities (the *variance risk premium*) and we find that a positive variance risk premium (an anticipated increase in variance) has more impact on returns than a negative variance risk premium.

Keywords: Volatility asymmetry, leverage effect, volatility feedback effect, risk premium, variance risk premium, multi-horizon causality, causality measure, high-frequency data, realized volatility, bipower variation, implied volatility.

Journal of Economic Literature classification: G1; G12; G14; C1; C12; C15; C32; C51; C53.

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1. Introduction

One of the many stylized facts about equity returns is an asymmetric relationship between returns and volatility. Volatility tends to rise following negative returns and fall following positive returns. Two main explanations for volatility asymmetry have been proposed in the literature. The first one is the *leverage effect*: a decrease in the price of an asset increases financial leverage and the probability of bankruptcy, making the asset riskier, hence an increase in volatility; see Black (1976) and Christie (1982). When applied to an equity index, this original idea translates into a dynamic leverage effect.[1] The second explanation is the *volatility feedback effect*, which is related to a time-varying risk premium: if volatility is priced, an anticipated increase in volatility raises the required rate of return, implying an immediate stock price decline in order to allow for higher future returns; see Pindyck (1984), French, Schwert and Stambaugh (1987), Campbell and Hentschel (1992), and Bekaert and Wu (2000).

In this paper, we provide new evidence on these two mechanisms of interaction between returns and volatilities by considering causality measures on high-frequency data. We also stress the importance of distinguishing between *realized volatility* and *implied volatility* when studying leverage and volatility feedback effects, and we find that *implied volatilities* are essential for assessing the *volatility feedback effect*.

On noting that the two explanations involve different *causal* mechanisms [see Bekaert and Wu (2000) and Bollerslev et al. (2006)], which may differ both through their direction and the time lags involved, we study the issue using short and long-run causality measures recently introduced in Dufour and Taamouti (2009). The causality measures allow us to study and test the asymmetric volatility phenomena at several horizons. When considering horizons longer than one period, it is important to account for indirect causality. Auxiliary variables can transmit causality between two variables of interest at horizons strictly higher than one, even if there is no causality between the two variables at the horizon one; see Dufour and Renault (1998). Using high-frequency data increases the chance to detect causal links since aggregation may make the relationship between returns and volatility simultaneous. By relying on realized volatility measures we avoid the need to specify a volatility model.

To be more explicit on the *causality* issue involved, the leverage effect explains why a negative return shock leads to higher subsequent volatility, while the volatility feedback effect explains how

[1]The concept of leverage effect, which means that negative returns today increases volatility of tomorrow, was introduced for individual stocks (or firms). However, it has also been applied to stock market indices; see Bouchaud, Matacz and Potters (2001), Jacquier, Polson and Rossi (2004), Brandt and Kang (2004), Ludvigson and Ng (2005), and Bollerslev, Litvinova and Tauchen (2006).

an anticipated increase in volatility may result in a negative return. Thus, volatility asymmetry may result from various causal links: from returns to volatility, from volatility to returns, instantaneous causality. Causality here is defined as in Granger (1969): a variable Y causes a variable X if the variance of the forecast error of X obtained by using the past of Y is smaller than the variance of the forecast error of X obtained without using the past of Y. In order to quantify and compare the strength of dynamic leverage and volatility feedback effects, we propose to use vector autoregressive (VAR) models of returns and various measures of volatility at high frequency together with short and long-run causality measures in Dufour and Taamouti (2009).

Using 5-minute observations on S&P 500 Index futures contracts, we first consider causality measures based on a bivariate VAR involving returns and realized volatility. In this setting, we find a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility feedback effect appears to be negligible, irrespective of the horizon considered.

In studying the relationship between volatility and returns, *implied volatility* – derived from option prices – can be an interesting alternative measure of volatility or constitute a useful auxiliary variable, because option prices may capture additional relevant information as well as nonlinear relations. Implied volatility can be viewed as a forward-looking measure of volatility with an horizon corresponding to the maturity of the option. We find that adding implied volatility to the information set to forecast returns leads to statistical evidence for a sizable volatility feedback effect for a few days, whereas the leverage effect remains almost the same. A key element of the volatility feedback mechanism is an increase of expected future volatility. Implied volatility certainly provides an option market forecast of future volatility, which is better than a forecast based on past realized volatility. The informational content of implied volatility does not come as a surprise since several studies have documented that implied volatility can be used to predict whether a market is likely to move higher or lower and help to predict future volatility; see Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Fleming (1998), Poteshman (2000), Blair, Poon and Taylor (2001), and Busch, Christensen and Nielsen (2006). Pooling the information contained in futures and options markets unveils an effect that cannot be found with one market alone. This is a new and important empirical finding.

Another contribution of this paper consists in showing that the proposed causality measures help to quantify the dynamic impact of bad and good return news on volatility.[2] A common approach

[2] In this study bad and good news are determined by negative and positive innovations in returns and volatility. Another literature considers the impact of macroeconomic news announcements on financial markets (e.g. volatility), see for example Cutler, Poterba and Summers (1989), Schwert (1981), Pearce and Roley (1985), Hardouvelis (1987), Haugen,

for empirically visualizing the relationship between news and volatility is provided by the news-impact curve originally studied by Pagan and Schwert (1990) and Engle and Ng (1993). To study the effect of current return shocks on future expected volatility, Engle and Ng (1993) introduced the News Impact Function (hereafter *NIF*). The basic idea of this function is to consider the effect of the return shock at time t on volatility at time $t+1$ in isolation while conditioning on information available at time t and earlier. Engle and Ng (1993) explain that this curve, where all the lagged conditional variances are evaluated at the level of the asset return unconditional variance, relates past positive and negative returns to current volatility.

We propose a new curve, the *Causal News Impact Function* (*CNIF*), for capturing the impact of news on volatility based on causality measures. In contrast with the *NIF* of Engle and Ng (1993), the CNIF curve can be constructed for parametric and stochastic volatility models and it allows one to consider all the past information about volatility and returns. We also build confidence intervals using a bootstrap technique around the CNIF curve. Further, we can visualize the impact of news on volatility at different horizons [see also Chen and Ghysels (2007)] rather than only one horizon as in Engle and Ng (1993).

We confirm by simulation that the CNIF based on causality measures detects well the differential effect of good and bad news in various parametric volatility models. Then, we apply the concept to the S&P 500 Index futures returns and volatility: we find a much stronger impact from bad news at several horizons. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons.

Our results on the informational value of implied volatility also suggest that the difference between implied and realized volatility (called the *variance risk premium*) constitutes an interesting measure of "news" coming to the market. So we compute causality measures from positive and negative variance risk premia to returns. We find a stronger impact when the difference is positive (an anticipated increase in volatility or bad news) than when it is negative.

Recently, two studies have used high-frequency data to study the relationship between returns and volatility. Using high-frequency data and simple *correlations*, Bollerslev et al. (2006) find an important negative correlation between volatility and current and lagged returns lasting for several days, while correlations between returns and lagged volatility are all close to zero. Masset and Martin (2008) use high-frequency data to analyze the lead-lag relationship of option implied volatility and index return in Germany based on Granger causality tests and impulse-response func-

Talmor and Torous (1991), Jain (1988), McQueen and Roley (1993), Balduzzi, Elton and Green (2001), Andersen, Bollerslev, Diebold and Vega (2003), and Huang (2007).

tions. They find that the relationship is return-driven in the sense that index returns Granger cause volatility changes. An important difference between our paper and Bollerslev et al. (2006) and Masset and Martin (2008) papers is that, among other things, we show that *implied volatilities* are important for assessing the *volatility feedback effect*. Further, in the present paper we use short and long-run causality measures to quantify the causality at different horizons, whereas in their papers they consider simple correlations and impulse-response functions which are inappropriate measures of causality: to see why impulse-response functions are inappropriate measures of causality, the reader can consult Dufour and Renault (1998).

Previous empirical evidence about the links between returns and volatility, often based on volatility models, is abundant but the messages about the sign of the relationship or about the prominence of the leverage effect or the volatility feedback effect are mixed. Studies focusing on the leverage hypothesis conclude that the latter cannot completely account for changes in volatility; see Christie (1982) and Schwert (1989). However, for the volatility feedback effect, empirical findings conflict. French et al. (1987), Campbell and Hentschel (1992) and Ghysels, Santa-Clara and Valkanov (2004) find a positive relation between volatility and expected returns, while Turner, Startz and Nelson (1989), Glosten, Jagannathan and Runkle (1993) and Nelson (1991) find a negative relation. Often the coefficient linking volatility to returns is statistically insignificant. Ludvigson and Ng (2005) find a strong positive contemporaneous relation between the conditional mean and conditional volatility and a strong negative lag-volatility-in-mean effect. Guo and Savickas (2006) conclude that the stock market risk-return relation is positive, as stipulated by the CAPM; however, idiosyncratic volatility is negatively related to future stock market returns.

Only a few studies have looked at the relation between returns and implied volatility [Giot (2005), Dennis, Mayhew and Stivers (2006), Bekaert and Wu (2000)]. These studies remain limited to relatively low frequency data (such as, daily data), do not take into account realized volatility (for which implied volatility may play the role of a *confounding factor*), and do not exploit the newer causal analysis framework used in the present paper. Giot (2005) uses S&P100 index and an implied volatility index (VIX) to show that there is a *contemporaneous* asymmetric relationship between S&P100 index returns and VIX: negative S&P100 index returns yield bigger changes in VIX than do positive returns [see Whaley (2000)]. He also assesses the possible relationship between implied volatility and forward looking stock index returns. He finds that there is some evidence that positive (negative) forward looking returns are to be expected for long positions in the stock index triggered by extremely high (low) levels of the implied volatility indices. Dennis et al. (2006), using daily stock returns and innovations in option-derived implied volatilities, show that the relation between stock returns and innovations in systematic volatility (idiosyncratic volatility) is substantially negative (near zero). These results suggest that asymmetric volatility is primarily attributed to systematic influences (such as feedback of market-level volatility changes), rather than aggregated firm-level effects (such as leverage). For individual assets, Bekaert and Wu (2000) argue that the volatility feedback effect dominates the leverage effect empirically.

The plan of the paper is as follows. In Section 2, we define volatility measures in high-frequency data and we review the concept of causality at different horizons and its measures. In Section 3, we propose and discuss VAR models that allow us to measure leverage and volatility feedback effects with high-frequency data. In Section 4, we propose to use implied volatility (IV) – in addition to realized volatility and returns – in order to measure the dynamic leverage and volatility feedback effects. Section 5 describes the high-frequency data, the estimation procedure and the empirical findings regarding causality effects between volatility and returns. In Section 6, we propose a method to assess the dynamic impact of good and bad return news on volatility. Simulation results on the efficiency of this method are also presented. Our empirical results on news effects in S&P 500 futures market appear in Section 7. We conclude in Section 8.

2. Volatility and causality measures

To assess causality between volatility and returns at high frequency, we need to build measures for both volatility and causality. For volatility, we use various measures of realized volatility introduced by Andersen, Bollerslev and Diebold (2003*a*); see also Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold and Labys (2001), Barndorff-Nielsen and Shephard (2002*a*), and Barndorff-Nielsen and Shephard (2002*b*). For causality, we rely on the short and long run causality measures proposed by Dufour and Taamouti (2009).

Let us first set some notations. We denote the time-t logarithmic price of the risky asset or portfolio by p_t and the continuously compounded returns from time t to $t+1$ by $r_{t+1} = p_{t+1} - p_t$. We assume that the price process may exhibit both stochastic volatility and jumps. It could belong to the class of continuous-time jump diffusion processes,

$$dp_t = \mu_t dt + \sigma_t dW_t + \kappa_t dq_t, \ \ 0 \leq t \leq T, \tag{2.1}$$

where μ_t is a continuous and locally bounded variation process, σ_t is the stochastic volatility process, W_t denotes a standard Brownian motion, dq_t is a counting process with $dq_t = 1$ corresponding to a jump at time t and $dq_t = 0$ otherwise, with jump intensity λ_t. The parameter κ_t refers to the size of the corresponding jumps. Thus, the quadratic variation of returns from time t to $t+1$ is given by

$$[r,\, r]_{t+1} = \int_t^{t+1} \sigma_s^2 ds + \sum_{0<s\leq t} \kappa_s^2 \tag{2.2}$$

where the first component, called integrated volatility, comes from the continuous component of (2.1), and the second term is the contribution from discrete jumps. In the absence of jumps, the second term on the right-hand-side disappears, and the quadratic variation is simply equal to the integrated volatility.

2.1. Volatility in high-frequency data: realized volatility, bipower variation, jumps

In this section, we define the various high-frequency measures that we will use to capture volatility. In what follows we normalize the daily time-interval to unity and we divide it into h periods. Each period has length $\Delta = 1/h$. Let the discretely sampled Δ-period returns be denoted by $r_{(t,\,\Delta)} = p_t - p_{t-\Delta}$ and the daily return by $r_{t+1} = \sum_{j=1}^{h} r_{(t+j.\Delta,\,\Delta)}$. The daily realized volatility is defined as the summation of the corresponding h high-frequency intradaily squared returns:

$$RV_{t+1} \equiv \sum_{j=1}^{h} r^2_{(t+j\Delta,\,\Delta)}. \tag{2.3}$$

The realized volatility satisfies

$$\lim_{\Delta \longrightarrow 0} RV_{t+1} = \int_t^{t+1} \sigma_s^2 ds + \sum_{0<s\leq t} \kappa_s^2\,, \tag{2.4}$$

which means that RV_{t+1} is a consistent estimator of the sum of the integrated variance $\int_t^{t+1} \sigma_s^2 ds$ and the jump contribution; see Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold and Labys (2001), Andersen, Bollerslev and Diebold (2003*a*), Barndorff-Nielsen and Shephard (2002*a*, 2002*b*), and Comte and Renault (1998).[3] Similarly, a measure of standardized bipower variation is given by

$$BV_{t+1} \equiv \frac{\pi}{2} \sum_{j=2}^{h} \mid r_{(t+j\Delta,\,\Delta)} \mid\mid r_{(t+(j-1)\Delta,\,\Delta)} \mid\ . \tag{2.5}$$

Under reasonable assumptions on the dynamics of (2.1), the bipower variation satisfies

$$\lim_{\Delta \longrightarrow 0} BV_{t+1} = \int_t^{t+1} \sigma_s^2 ds\,; \tag{2.6}$$

[3]For a general discussion of integrated and realized volatilities in the absence of jumps, see Meddahi (2002).

see Barndorff-Nielsen and Shephard (2004) and Barndorff-Nielsen, Graversen, Jacod, Podolskij and Shephard (2005). Equation (2.6) means that BV_{t+1} provides a consistent estimator of the integrated variance unaffected by jumps. Finally, as noted by Barndorff-Nielsen and Shephard (2004), combining the results in equation (2.4) and (2.6), the contribution to the quadratic variation due to the discontinuities (jumps) in the underlying price process may be consistently estimated by

$$\lim_{\Delta \longrightarrow 0} (RV_{t+1} - BV_{t+1}) = \sum_{0<s\leq t} \kappa_s^2. \tag{2.7}$$

We can also define the relative measure

$$RJ_{t+1} = \frac{(RV_{t+1} - BV_{t+1})}{RV_{t+1}} \tag{2.8}$$

or the corresponding logarithmic ratio

$$\bar{J}_{t+1} = \ln(RV_{t+1}) - \ln(BV_{t+1}). \tag{2.9}$$

Huang and Tauchen (2005) argue that these are more robust measures of the contribution of jumps to total price variation. Since in practice J_{t+1} can be negative in a given sample, we impose a non-negativity truncation of the actual empirical jump measurements:

$$J_{t+1} \equiv \max[\ln(RV_{t+1}) - \ln(BV_{t+1}),\, 0]\,; \tag{2.10}$$

see Andersen, Bollerslev and Diebold (2003*a*) and Barndorff-Nielsen and Shephard (2004).

2.2. Short-run and long-run causality measures

We study the causality at different horizons between returns (r_t) and volatilities (σ_t^2). For that purpose, it will be convenient to define first *noncausality* in terms of orthogonality between subspaces of a Hilbert space of random variables with finite second moments. To give a formal definition of noncausality at different horizons, we need to consider the following notations. We denote by $r(\omega, t],\ \sigma^2(\omega, t],$ and $z(\omega, t]$ the information contained in the history of variables of interest r and σ^2 and another auxiliary variable z respectively up to time t. The "starting point" ω is typically equal to a finite initial date (such as $\omega = -1,\ 0$ or 1) or to $-\infty$. In our empirical application the auxiliary variable z is given by the implied volatility (hereafter IV). The information sets obtained by "adding" $z(\omega, t]$ to $r(\omega, t]$, $z(\omega, t]$ to $\sigma^2(\omega, t]$, $r(\omega, t]$ to $\sigma^2(\omega, t]$, and $z(\omega, t]$ to $r(\omega, t]$ and

$\sigma^2(\omega, t]$ are defined as:

$$I_{rz}(t) = I_0 + r(\omega,\, t] + z(\omega,\, t]\,,\;\; I_{\sigma^2 z}(t) = I_0 + \sigma^2(\omega,\, t] + z(\omega,\, t]\,, \tag{2.11}$$

$$I_{r\sigma^2}(t) = I_0 + r(\omega,\, t] + \sigma^2(\omega,\, t]\,,\;\; I_{r\sigma^2 z}(t) = I_0 + r(\omega,\, t] + \sigma^2(\omega,\, t] + z(\omega,\, t]\,, \tag{2.12}$$

where I_0 represents a fundamental information set available in all cases (such as deterministic variables, a constant, etc.). Finally, for any given information set B_t, we denote by $\mathsf{Var}[r_{t+h} \mid B_t]$ (respectively $\mathsf{Var}[\sigma^2_{t+h} \mid B_t]$) the variance of the forecast error of r_{t+h} $\big($respectively $\sigma^2_{t+h}\big)$ based on the information set B_t.[4] Thus, we have the following definition of noncausality at different horizons [see Dufour and Renault (1998) and Dufour and Taamouti (2009)].

Definition 2.1 *Let h be a positive integer.*
(i) r does not cause σ^2 at horizon h given $I_{\sigma^2 z}(t)$, denoted $r \underset{h}{\nrightarrow} \sigma^2 \mid I_{\sigma^2 z}(t)$, iff

$$\mathsf{Var}\left[\sigma^2_{t+h} \mid I_{\sigma^2 z}(t)\right] = \mathsf{Var}\left[\sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)\right]\;; \tag{2.13}$$

(ii) r does not cause σ^2 up to horizon h given $I_{\sigma^2 z}(t)$, denoted $r \underset{(h)}{\nrightarrow} \sigma^2 \mid I_{\sigma^2 z}(t)$, iff

$$r \underset{k}{\nrightarrow} \sigma^2 \mid I_{\sigma^2 z}(t) \text{ for } k = 1,\, 2,\, \ldots\,,\, h\,; \tag{2.14}$$

(iii) r does not cause σ^2 at any horizon given $I_{\sigma^2 z}(t)$, denoted $r \underset{(\infty)}{\nrightarrow} \sigma^2 \mid I_{\sigma^2 z}(t)$, iff

$$r \underset{k}{\nrightarrow} \sigma^2 \mid I_{\sigma^2 z}(t) \text{ for all } k = 1,\, 2,\, \ldots \tag{2.15}$$

Definition **2.1** corresponds to causality from r to σ^2 and means that r causes σ^2 at horizon h if the past of r improves the forecast of σ^2_{t+h} given the information set $I_{\sigma^2 z}(t)$. We can similarly define noncausality at horizon h from σ^2 to r. The presence of auxiliary variable z may transmit the causality between r and σ^2 at horizon h strictly higher than one even if there is no causality between the two variables at horizon 1. However, in the absence of auxiliary variable, noncausality at horizon 1 implies noncausality at any horizon h strictly higher than one; see Dufour and Renault (1998). In other words,

$$r \underset{1}{\nrightarrow} \sigma^2 \mid \sigma^2(\omega,\, t] \;\Rightarrow\; r \underset{(\infty)}{\nrightarrow} \sigma^2 \mid I_{\sigma^2}(t)\,, \tag{2.16}$$

$$\sigma^2 \underset{1}{\nrightarrow} r \mid r(\omega,\, t] \;\Rightarrow\; \sigma^2 \underset{(\infty)}{\nrightarrow} r \mid I_r(t)\,, \tag{2.17}$$

[4] B_t can be equal to $I_{r\sigma z}(t)$, $I_{rz}(t)$, or $I_{\sigma z}(t)$.

where $I_{\sigma^2}(t) = I_0 + \sigma^2(\omega, t]$ and $I_r(t) = I_0 + r(\omega, t]$. A measure of causality from r to σ^2 at horizon $h,$ denoted $C(r \underset{h}{\longrightarrow} \sigma^2),$ is given by following function [see Dufour and Taamouti (2009)]:

$$C(r \underset{h}{\longrightarrow} \sigma^2) = \ln \left[\frac{\mathsf{Var}[\sigma^2_{t+h} \mid I_{\sigma^2 z}(t)]}{\mathsf{Var}[\sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)]} \right]. \tag{2.18}$$

Similarly, a measure of causality from σ^2 to r at horizon $h,$ denoted $C(\sigma^2 \underset{h}{\longrightarrow} r),$ is given by:

$$C(\sigma^2 \underset{h}{\longrightarrow} r) = \ln \left[\frac{\mathsf{Var}[r_{t+h} \mid I_{rz}(t)]}{\mathsf{Var}[r_{t+h} \mid I_{r\sigma^2 z}(t)]} \right]. \tag{2.19}$$

For example, $C(r \underset{h}{\longrightarrow} \sigma^2)$ measures the causal effect from r to σ^2 at horizon h given the past of σ^2 and z. In terms of predictability, it measures the information given by the past of r that can improve the forecast of σ^2_{t+h}. Since $\mathsf{Var}[\sigma^2_{t+h} \mid I_{\sigma^2 z}(t)] \geq \mathsf{Var}[\sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)],$ the function $C(r \underset{h}{\longrightarrow} \sigma^2)$ is nonnegative. Furthermore, it is zero when when there is no causality at horizon h. However, as soon as there is causality at horizon 1, causality measures at different horizons may considerably differ.

In Dufour and Taamouti (2009), a measure of instantaneous causality between r and σ^2 at horizon h is also proposed. It is given by the function

$$C(r \underset{h}{\leftrightarrow} \sigma^2) = \ln \left[\frac{\mathsf{Var}[r_{t+h} \mid I_{r\sigma^2 z}(t)]\, \mathsf{Var}[\sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)]}{\det\left(\Sigma\left[r_{t+h},\, \sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)\right]\right)} \right] \tag{2.20}$$

where $\det\left(\Sigma\left[r_{t+h},\, \sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)\right]\right)$ represents the determinant of the variance-covariance matrix $\Sigma\left[r_{t+h},\, \sigma^2_{t+h} \mid I_{r\sigma^2 z}(t)\right]$ of the forecast error of the joint process $\left(r,\, \sigma^2\right)'$ at horizon h given the information set $I_{r\sigma^2 z}(t)$. Note that σ^2 may be replaced by $\ln(\sigma^2)$. Since the logarithmic transformation is nonlinear, this may modify the value of the causality measure.

In what follows, we apply the above measures to study the causality at different horizons from returns to volatility (hereafter leverage effect), from volatility to returns (hereafter volatility feedback effect), and the instantaneous causality and dependence between returns and volatility. In Section 3, we study these effects by considering a limited information set which contains only the past of returns and realized volatility. In Section 4, we extended our information set by adding the past of implied volatility.

3. Measuring leverage and volatility feedback effects in a VAR model

In this section, we study the relationship between the return r_t and its volatility σ_t^2. The objective is to measure and compare the strength of *dynamic* leverage and volatility feedback effects in high-frequency equity data. These effects are quantified within the context of a VAR model and by using short and long run causality measures proposed by Dufour and Taamouti (2009). Since the volatility asymmetry may be the result of causality from returns to volatility [leverage effect], from volatility to returns [volatility feedback effect], instantaneous causality, all of these causal effects, or some of them. We wish to measure all these effects and to compare them in order to determine the most important ones.

We suppose that the joint process of returns and logarithmic volatility, $(r_{t+1}, \ln(\sigma_{t+1}^2))'$ follows an autoregressive linear model

$$\begin{pmatrix} r_{t+1} \\ \ln(\sigma_{t+1}^2) \end{pmatrix} = \mu + \sum_{j=1}^{p} \Phi_j \begin{pmatrix} r_{t+1-j} \\ \ln(\sigma_{t+1-j}^2) \end{pmatrix} + u_{t+1} \tag{3.1}$$

where

$$\mu = \begin{pmatrix} \mu_r \\ \mu_\sigma \end{pmatrix}, \; u_{t+1} = \begin{pmatrix} u_{t+1}^r \\ u_{t+1}^\sigma \end{pmatrix}, \quad \Phi_j = \begin{bmatrix} \Phi_{11j} & \Phi_{12j} \\ \Phi_{21j} & \Phi_{22j} \end{bmatrix}, \; j = 1, \ldots, p, \tag{3.2}$$

$$\mathsf{E}\left[u_t\right] = 0 \text{ and } \mathsf{E}\left[u_t u_s'\right] = \begin{cases} \Sigma_u & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases}. \tag{3.3}$$

In the empirical application σ_{t+1}^2 will be replaced by the realized volatility RV_{t+1} or the bipower variation BV_{t+1}. The disturbance u_{t+1}^r is the one-step-ahead error when r_{t+1} is forecast from its own past and the past of $\ln(\sigma_{t+1}^2)$, and similarly u_{t+1}^σ is the one-step-ahead error when $\ln(\sigma_{t+1}^2)$ is forecast from its own past and the past of r_{t+1}. We suppose that these disturbances are each serially uncorrelated, but may be correlated with each other contemporaneously and at various leads and lags. Since u_{t+1}^r is uncorrelated with $I_{r\sigma^2}(t)$, the equation for r_{t+1} represents the linear projection of r_{t+1} on $I_{r\sigma^2}(t)$. Likewise, the equation for $\ln(\sigma_{t+1}^2)$ represents the linear projection of $\ln(\sigma_{t+1}^2)$ on $I_{r\sigma^2}(t)$.

Equation (3.1) allows one to model the first two conditional moments of the asset returns. We model conditional volatility as an exponential function process to guarantee that it is positive. The first equation of the $VAR(p)$ in (3.1) describes the dynamics of the return as

$$r_{t+1} = \mu_r + \sum_{j=1}^{p} \Phi_{11j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{12j} \ln(\sigma_{t+1-j}^2) + u_{t+1}^r. \tag{3.4}$$

This equation allows to capture the temporary component of Fama and French (1988) permanent and temporary components model, in which stock prices are governed by a random walk and a stationary autoregressive process, respectively. For $\Phi_{12j} = 0$, this model of the temporary component is the same as that of Lamoureux and Lastrapes (1993); see also Brandt and Kang (2004), and Whitelaw (1994). The second equation of $VAR(p)$ describes the volatility dynamics as

$$\ln(\sigma^2_{t+1}) = \mu_\sigma + \sum_{j=1}^{p} \Phi_{21j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{22j} \ln(\sigma^2_{t+1-j}) + u^\sigma_{t+1}, \tag{3.5}$$

and it represents the standard stochastic volatility model. For $\Phi_{21j} = 0$, equation (3.5) can be viewed as the stochastic volatility model estimated by Wiggins (1987) , Andersen and Sørensen (1996), and many others. However, in this paper we consider that σ^2_{t+1} is not a latent variable and it can be approximated by realized or bipower variations from high-frequency data. We also note that the conditional mean equation includes the volatility-in-mean model used by French et al. (1987) and Glosten et al. (1993) to explore the contemporaneous relationship between the conditional mean and volatility [see Brandt and Kang (2004)]. To illustrate the connection to the volatility-in-mean model, we premultiply the system in (3.1) by the matrix

$$P = \begin{bmatrix} 1 & -\frac{\mathsf{Cov}(r_{t+1},\ \ln(\sigma^2_{t+1}))}{\mathsf{Var}[\ln(\sigma^2_{t+1})|I_{r\sigma^2}(t)]} \\ -\frac{\mathsf{Cov}(r_{t+1},\ \ln(\sigma^2_{t+1}))}{\mathsf{Var}[r_{t+1}|I_{r\sigma^2}(t)]} & 1 \end{bmatrix}. \tag{3.6}$$

Then, the first equation of r_{t+1} is a linear function of the elements of $r(\omega, t]$, $\sigma^2(\omega, t+1]$, and the disturbance $u^r_{t+1} - \frac{\mathsf{Cov}(r_{t+1},\ \ln(\sigma^2_{t+1}))}{\mathsf{Var}[\ln(\sigma^2_{t+1})|I_{r\sigma^2}(t)]}\, u^\sigma_{t+1}$. Since this disturbance is uncorrelated with u^σ_{t+1}, it is uncorrelated with $\ln(\sigma^2_{t+1})$ as well as with $r(\omega, t]$ and $\sigma^2(\omega, t+1]$. Hence the linear projection of r_{t+1} on $r(\omega, t]$ and $\sigma^2(\omega, t+1]$ is provided by the first equation of the new system:

$$r_{t+1} = \nu_r + \sum_{j=1}^{p} \phi_{11j} r_{t+1-j} + \sum_{j=0}^{p} \phi_{12j} \ln(\sigma^2_{t+1-j}) + \tilde{u}^r_{t+1}\,. \tag{3.7}$$

The new parameters $\nu_r,\ \phi_{11j}$, and ϕ_{12j}, for $j = 0,\ 1, \ldots, p$, are functions of parameters in the vector μ and matrix Φ_j, for $j = 1, \ldots, p$. Equation (3.7) is a generalized version of the usual volatility-in-mean model, in which the conditional mean depends contemporaneously on the conditional volatility. Similarly, the existence of the linear projection of $\ln(\sigma^2_{t+1})$ on $r(\omega, t+1]$ and $\sigma^2(\omega, t]$,

$$\ln(\sigma^2_{t+1}) = \nu_\sigma + \sum_{j=0}^{p} \phi_{21j} r_{t+1-j} + \sum_{j=1}^{p} \phi_{22j} \ln(\sigma^2_{t+1-j}) + \tilde{u}^\sigma_{t+1} \tag{3.8}$$

follows from the second equation of the new system. The new parameters $\nu_\sigma,\ \phi_{21j},$ and $\phi_{22j},$ for $j = 1, \ldots, p,$ are functions of parameters in the vector μ and matrix $\Phi_j,$ for $j = 1, \ldots, p$. The volatility model given by equation (3.8) captures the persistence of volatility through the terms ϕ_{22j}. In addition, it incorporates the effects of the mean on volatility, both at the contemporaneous and intertemporal levels through the coefficients ϕ_{21j}, for $j = 0,\ 1, \ldots, p$.

Let us now consider the matrix

$$\Sigma_u = \begin{bmatrix} \sigma^2_{u^r} & c \\ c & \sigma^2_{u^\sigma} \end{bmatrix}, \tag{3.9}$$

where $\sigma^2_{u^r}$ and $\sigma^2_{u^\sigma}$ represent the variances of the one-step-ahead forecast errors of return and volatility, respectively. c represents the covariance between these errors. Based on system (3.1), the forecast error of $(r_{t+h}, \ln(\sigma^2_{t+h}))'$ is given by:

$$e\left[\left(r_{t+h},\ \ln(\sigma^2_{t+h})\right)'\right] = \sum_{i=0}^{h-1} \psi_i u_{t+h-i}, \tag{3.10}$$

where the coefficients $\psi_i,$ for $i = 0, \ldots, h-1$, represent the impulse response coefficients of the $MA(\infty)$ representation of model (3.1). These coefficients are given by the following equations:

$$\begin{aligned}
&\psi_0 = I\,,\\
&\psi_1 = \Phi_1\psi_0 = \Phi_1\,,\\
&\psi_2 = \Phi_1\psi_1 + \Phi_2\psi_0 = \Phi_1^2 + \Phi_2\,,\\
&\psi_3 = \Phi_1\psi_2 + \Phi_2\psi_1 + \Phi_2\psi_0 = \Phi_1^3 + \Phi_1\Phi_2 + \Phi_2\Phi_1 + \Phi_3\,,\\
&\vdots
\end{aligned} \tag{3.11}$$

where I is an identity matrix and

$$\Phi_j = 0\,, \ \text{ for } j \geq p+1.$$

The covariance matrix of the forecast error (3.10) is given by

$$\mathsf{Var}\left[e[\left(r_{t+h},\ \ln(\sigma^2_{t+h})\right)']\right] = \sum_{i=0}^{h-1} \psi_i\ \Sigma_u \psi_i'. \tag{3.12}$$

We also consider the following restricted model:

$$\begin{pmatrix} r_{t+1} \\ \ln(\sigma^2_{t+1}) \end{pmatrix} = \bar{\mu} + \sum_{j=1}^{\bar{p}} \bar{\Phi}_j \begin{pmatrix} r_{t+1-j} \\ \ln(\sigma^2_{t+1-j}) \end{pmatrix} + \bar{u}_{t+1} \tag{3.13}$$

where

$$\bar{\mu} = \begin{pmatrix} \bar{\mu}_r \\ \bar{\mu}_\sigma \end{pmatrix}, \ \bar{u}_{t+1} = \begin{pmatrix} \bar{u}^r_{t+1} \\ \bar{u}^\sigma_{t+1} \end{pmatrix}, \ \bar{\Phi}_j = \begin{bmatrix} \bar{\Phi}_{11j} & 0 \\ 0 & \bar{\Phi}_{22j} \end{bmatrix}, \ j = 1, .., \bar{p}, \tag{3.14}$$

$$\mathsf{E}\left[\bar{u}_t\right] = 0\,, \ \mathsf{E}\left[\bar{u}_t\bar{u}_s^{'}\right] = \begin{cases} \bar{\Sigma}_u & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases}, \ \bar{\Sigma}_u = \begin{bmatrix} \Sigma_{\bar{u}^r} & \bar{c} \\ \bar{c} & \Sigma_{\bar{u}^\sigma} \end{bmatrix}. \tag{3.15}$$

Zero values in $\bar{\Phi}_j$ mean that there is noncausality at horizon 1 from returns to volatility and from volatility to returns. As mentioned in subsection 2.2, in a bivariate system, noncausality at horizon one implies noncausality at any horizon h strictly higher than one. This means that the absence of leverage effect at horizon one (respectively the absence of volatility feedback effect at horizon one) which corresponds to $\bar{\Phi}_{21j} = 0,$ for $j = 1, \ldots, \bar{p},$ (respectively $\bar{\Phi}_{12j} = 0,$ for $j = 1, \ldots, \bar{p},$) is equivalent to the absence of leverage effect (respectively volatility feedback effect) at any horizon $h \geq 1$.

To compare the forecast error variance of model (3.1) with that of model (3.13), we assume that $p = \bar{p}$. Based on the restricted model (3.13), the covariance matrix of the forecast error of $(r_{t+h}, \ln(\sigma^2_{t+h}))^{'}$ is given by:

$$\overline{\mathsf{Var}}\left[\bar{e}[(r_{t+h},\, \ln(\sigma^2_{t+h}))^{'}]\right] = \sum_{i=0}^{h-1} \bar{\psi}_i\ \bar{\Sigma}_{\bar{u}} \bar{\psi}_i^{'}, \tag{3.16}$$

where the coefficients $\bar{\psi}_i,$ for $i = 0, \ldots, h-1,$ represent the impulse response coefficients of the $MA(\infty)$ representation of model (3.13). They can be calculated in the same way as in (3.11). From the covariance matrices (3.12) and (3.16), we define the following measures of leverage and volatility feedback effects at any horizon h, where $h \geq 1$,

$$C\big(r \underset{h}{\longrightarrow} \ln(\sigma^2)\big) = \ln\left[\frac{\sum_{i=0}^{h-1} e_2^{'}(\bar{\psi}_i\ \bar{\Sigma}_{\bar{u}} \bar{\psi}_i^{'})e_2}{\sum_{i=0}^{h-1} e_2^{'}(\psi_i\ \Sigma_u \psi_i^{'})e_2}\right], \ e_2 = (0,\, 1)^{'}, \tag{3.17}$$

$$C\big(\ln(\sigma^2) \underset{h}{\longrightarrow} r\big) = \ln\left[\frac{\sum_{i=0}^{h-1} e_1^{'}(\bar{\psi}_i\ \bar{\Sigma}_{\bar{u}} \bar{\psi}_i^{'})e_1}{\sum_{i=0}^{h-1} e_1^{'}(\psi_i\ \Sigma_u \psi_i^{'})e_1}\right], \ e_1 = (1,\, 0)^{'}. \tag{3.18}$$

The parametric measure of instantaneous causality at horizon h, where $h \geq 1,$ is given by the following function

$$C\big(r \underset{h}{\leftrightarrow} \ln(\sigma^2)\big) = \ln\left[\frac{(\sum_{i=0}^{h-1} e_2^{'}(\psi_i\ \Sigma_u \psi_i^{'})e_2)\ (\sum_{i=0}^{h-1} e_1^{'}(\psi_i\ \Sigma_u \psi_i^{'})e_1)}{\det(\sum_{i=0}^{h-1} \psi_i\ \Sigma_u \psi_i^{'})}\right]. \tag{3.19}$$

4. Implied volatility as an auxiliary variable

An important feature of causality is the information set considered to forecast the variables of interest. Until now, we have included only the past of returns and realized volatility. Since the volatility feedback effect rests on anticipating future movements in volatility it is natural to include option-based implied volatility, an all-important measure of market expectations of future volatility. Formally, we "add" the past of implied volatility to the information set $I_{r\sigma^2}(t)$ that we considered in the previous section. The new information set is given now by $I_{r\sigma^2 z}(t),$ where z is an auxiliary variable represented by implied volatility.

In this paper, we consider call options written on S&P 500 index futures contracts. The data come from the OptionMetrics data set which contains historical on option prices, dating back to January 1996. Given observations on the option price C and the remaining variables $S,\ K,\ \tau,$ and $r,$ an estimate of the implied volatility IV can be obtained by solving the nonlinear equation $C = C\left(S,\, K,\, \tau,\, r,\, IV^{1/2}\right)$ for $IV^{1/2}$, where $C(\cdot)$ refers to the Black-Scholes formula. Each day, we extract the implied volatility corresponding to the option that is closest to the money. This selection criterion ensures that the option will be liquid and therefore aggregates the opinion of many investors about future volatility. This appears more important than keeping a fixed maturity. This choice is often made in the empirical literature on option pricing [see for example Pan (2002)].

Therefore, we consider a trivariate autoregressive model including implied volatility, in addition to the realized volatility (bipower variation) and returns:[5]

$$\begin{bmatrix} r_{t+1} \\ RV^*_{t+1} \\ IV^*_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_r \\ \mu_{RV} \\ \mu_{IV} \end{bmatrix} + \sum_{j=1}^{p} \begin{bmatrix} \Phi_{11j} & \Phi_{12j} & \Phi_{13j} \\ \Phi_{21j} & \Phi_{22j} & \Phi_{23j} \\ \Phi_{31j} & \Phi_{32j} & \Phi_{33j} \end{bmatrix} \begin{bmatrix} r_{t+1-j} \\ RV^*_{t+1-j} \\ IV^*_{t+1-j} \end{bmatrix} + \begin{bmatrix} u^r_{t+1} \\ u^{RV}_{t+1} \\ u^{IV}_{t+1} \end{bmatrix} \tag{4.1}$$

where $RV^*_t = \ln(RV_t)$ and $IV^*_t = \ln(IV_t)$. The first equation of the above system

$$r_{t+1} = \mu_r + \sum_{j=1}^{p} \Phi_{11j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{12j} RV^*_{t+1-j} + \sum_{j=1}^{p} \Phi_{13j} IV^*_{t+1-j} + u^r_{t+1} \tag{4.2}$$

describes the dynamics of the return, while the second equation

$$RV^*_{t+1} = \mu_{RV} + \sum_{j=1}^{p} \Phi_{21j} r_{t+1-j} + \sum_{j=1}^{p} \Phi_{22j} RV^*_{t+1-j} + \sum_{j=1}^{p} \Phi_{23j} IV^*_{t+1-j} + u^{RV}_{t+1} \tag{4.3}$$

describes the volatility dynamics. It is well known that implied volatility can be used to predict

[5] Further, we consider an autoregressive model where we add jumps and our results do not change.

whether a market is likely to move higher or lower and help to predict future volatility [see Day and Lewis (1992), Canina and Figlewski (1993), Lamoureux and Lastrapes (1993), Poteshman (2000), Blair et al. (2001), and Busch et al. (2006)]. The forward-looking nature of the implied volatility measure makes it an ideal additional variable to capture a potential volatility feedback mechanism. Apart from using IV without any constraint in (4.2) and (4.3), we will also look at more restricted combinations dictated by financial considerations. Indeed, the difference between IV and RV provides an estimate of the risk premium attributable to the variance risk factor.

5. Causality measures for S&P 500 futures

In this section, we first describe the data used to measure causality in the VAR models of the previous sections. Then we explain how to estimate confidence intervals of causality measures for leverage and volatility feedback effects. Finally, we discuss our findings.

5.1. Data description

Our data consists of high-frequency tick-by-tick transaction prices for the S&P 500 Index futures contracts traded on the Chicago Mercantile Exchange, over the period January 1988 to December 2005 for a total of 4494 trading days. We eliminated a few days where trading was thin and the market was open for a shortened session. Due to the unusually high volatility at the opening, we also omit the first five minutes of each trading day [see Bollerslev et al. (2006)]. For reasons associated with microstructure effects we follow Bollerslev et al. (2006) and the literature in general and aggregate returns over five-minute intervals. We calculate the continuously compounded returns over each five-minute interval by taking the difference between the logarithm of the two tick prices immediately preceding each five-minute mark to obtain a total of 77 observations per day [see Müller, Dacorogna, Gençay, Olsen, and Pictet (2001) and Bollerslev et al. (2006) for more details]. We also construct hourly and daily returns by summing 11 and 77 successive five-minute returns, respectively.

Summary statistics for the five-minute, hourly, and daily returns and the associated volatilities are reported in tables 1 - 2 and displayed in figures 1-2 of Appendix B. From these, we see that the unconditional distributions of the returns exhibit high kurtosis and negative skewness. The sample kurtosis is much greater than the Gaussian value of three for all three series. The negative skewness remains moderate, especially for the five-minute and daily returns. Similarly, the unconditional distributions of realized and bipower volatility measures are highly skewed and leptokurtic. However, on applying a logarithmic transformation, both measures approximately normal [see Andersen, Bollerslev, Diebold and Ebens (2001)]. The descriptive statistics for the relative jump measure, J_{t+1}, clearly indicate a positively skewed and leptokurtic distribution. The time series plots of returns and volatilities show the familiar volatility clustering effect, along with some occasional large absolute returns.

It is also of interest to assess whether the realized and bipower volatility measures differ significantly. To test this, recall that

$$\lim_{\Delta \to 0} (RV_{t+1}) = \int_t^{t+1} \sigma_s^2 ds + \sum_{0<s\leq t} \kappa_s^2, \tag{5.1}$$

where $\int_t^{t+1} \sigma_s^2 ds$ is the integrated volatility and $\sum_{0<s\leq t} \kappa_s^2$ represents the contribution of jumps to total price variation. In the absence of jumps, the second term on the right-hand-side disappears, and the quadratic variation is simply equal to the integrated volatility: or asymptotically ($\Delta \to 0$) the realized variance is equal to the bipower variance. Many statistics have been proposed to test for the presence of jumps in financial data [see for example Barndorff-Nielsen and Shephard (2002*b*), Andersen, Bollerslev and Diebold (2003*b*), Huang and Tauchen (2005), among others]. In this paper, we test for the presence of jumps in our data by considering the following test statistics:

$$z_{QP,\,l,\,t} = \frac{RV_{t+1} - BV_{t+1}}{\sqrt{((\frac{\pi}{2})^2 + \pi - 5)\Delta QP_{t+1}}}\,, \tag{5.2}$$

$$z_{QP,\,t} = \frac{\ln(RV_{t+1}) - \ln(BV_{t+1})}{\sqrt{((\frac{\pi}{2})^2 + \pi - 5)\Delta \frac{QP_{t+1}}{BV_{t+1}^2}}}\,, \tag{5.3}$$

$$z_{QP,\,lm,\,t} = \frac{\ln(RV_{t+1}) - \ln(BV_{t+1})}{\sqrt{((\frac{\pi}{2})^2 + \pi - 5)\Delta \max(1,\, \frac{QP_{t+1}}{BV_{t+1}^2})}}\,, \tag{5.4}$$

where QP_{t+1} is the realized Quad-Power Quarticity [Barndorff-Nielsen and Shephard (2002*a*)], with

$$QP_{t+1} = h\mu_1^{-4} \sum_{j=4}^{h} \mid r_{(t+j.\Delta,\,\Delta)} \mid\mid r_{(t+(j-1).\Delta,\,\Delta)} \mid\mid r_{(t+(j-2).\Delta,\,\Delta)} \mid\mid r_{(t+(j-3).\Delta,\,\Delta)} \mid, \tag{5.5}$$

and $\mu_1 = \sqrt{\frac{2}{\pi}}$. Under the assumption of no jumps and for each time t, the statistics $z_{QP,\,l,\,t}$, $z_{QP,\,t}$, and $z_{QP,\,lm,\,t}$ follow a Normal distribution $\mathcal{N}(0,1)$ as $\Delta \to 0$. The results of testing for jumps in our data are plotted in Figure 3 of Appendix B. These graphs represent the quantile to quantile plots (hereafter QQ plot) of the relative measure of jumps given by equation (2.8) and the QQ Plots of the other statistics; $z_{QP,\,l,\,t}$, $z_{QP,\,t}$, and $z_{QP,\,lm,\,t}$. When there are no jumps, we expect that the cross line and the dotted line in Figure 3 will coincide. However, as this figure shows, the two lines are clearly distinct, indicating the presence of jumps in our data. Therefore, we will present our results for both realized volatility and bipower variation.

5.2. Causality measures

We examine several empirical issues regarding the relationship between volatility and returns. Before high-frequency data were not available and the concept of realized volatility took root – such issues could only be addressed through volatility models. Recently, Bollerslev et al. (2006) looked at these relationships using high-frequency data and realized volatility measures. As they emphasize, the fundamental difference between the leverage and the volatility feedback explanations lies in the direction of causality. The leverage effect explains why a low return causes higher subsequent volatility, while the volatility feedback effect captures how an increase in volatility may cause a negative return. However, they studied only correlations between returns and volatility at various leads and lags, not causality relationships.

Here, we apply short-run and long-run causality measures to quantify the strength of the relationships between return and volatility. We use OLS to estimate the VAR(p) models described above and the Akaike information criterion to specify their orders.[6] To obtain consistent estimates of the causality measures, we simply replace the unknown parameters by their estimates. We calculate causality measures for various horizons $h = 1, \ldots, 20$. A higher value for a causality measure indicates a stronger causality. We also compute the corresponding nominal 95% bootstrap percentile confidence intervals according to the procedure described in Appendix A. As mentioned by Inoue and Kilian (2002), for bounded measures, as in our case, the bootstrap approach is more reliable than the delta-method. One reason is because the delta-method interval is not range respecting and may produce confidence intervals that are logically invalid. In contrast, the bootstrap percentile interval preserves by construction these constraints [see Inoue and Kilian (2002, pages 315-318) and Efron and Tibshirani (1993)]. Further, the percentile interval allows avoiding using the variance-covariance matrix of the estimators which depends on the homoskedasticity assumption. More details on the consistency and statistical justification of the procedures used here are available in Dufour and Taamouti (2009).

The concept of Granger causality requires an information set and is analyzed in the framework of a model between the variables of interest. Both the strength of this causal link and its statistical

[6] Using Akaike's criterion we find that the appropriate value of the order of the unconstrained autoregressive model is equal to 10. Since using the same criterion the value of the order of the constrained model is smaller than 10, we take $p = \bar{p} = 10$ [see Section 3].

significance are important. A major obstacle to detecting causality is aggregation. Low frequency data may mask the true causal relationship between variables. High-frequency data thus offer an opportunity to analyze causal effects. In particular, we can distinguish with an exceptionally high resolution between immediate and lagged effects. Further, even if one's interest focuses on relationships at the daily frequency, using higher-frequency data to construct daily returns and volatilities can provide better estimates than using daily returns (as done in previous studies). Besides, since measured realized volatility can be viewed as an approximation to the "true" unobservable volatility, we consider both raw realized volatility and the bipower variation (which provides a way to filter out possible jumps in the data); see Barndorff-Nielsen and Shephard (2004).

With five-minute intervals we could estimate the VAR model at this frequency. However, if we wanted to allow for enough time for the effects to develop we would need a large number of lags in the VAR model and sacrifice efficiency in the estimation. This problem arises in many studies of volatility forecasting. Researchers have use several schemes to group five-minute intervals, in particular the HAR-RV or the MIDAS schemes.[7] We decided to look both at hourly and daily frequencies.

Our empirical results will be presented mainly through graphs. Each figure reports the causality measure as a function of the horizon. The main results are summarized and compared in figures 4 - 7 of Appendix B. Detailed results, including confidence bands on the causality measures, are reported in Appendix C.

Results based on bivariate models indicate the following [Figure 4 and Table 3 in Appendix B and figures 11 - 12 in Appendix C]. When returns are aggregated to the hourly frequency, we find that the leverage effect is statistically significant for the first four hours, while the volatility feedback effect is negligible at all horizons. Using daily observations, derived from high-frequency data, we find a strong leverage effect for the first three days, while the volatility feedback effect appears to be negligible at all horizons. The results based on realized volatility (RV) and bipower variation (BV) are essentially the same [Figure 11 in Appendix C]. Overall, these results show that the leverage effect is more important than the volatility feedback effect [Figure 4 in Appendix B].

If the feedback effect from volatility to returns is almost-non-existent, it is apparent that the instantaneous causality between these variables exists and remains economically and statistically important for several days [see Figure 12 in Appendix C]. This means that volatility has a con-

[7]The HAR-RV scheme, in which the realized volatility is parameterized as a linear function of the lagged realized volatilities over different horizons has been proposed by Müller, Dacorogna, Davé, Olsen, Pictet and Von Weizsäcker (1997) and Corsi (2003). The MIDAS scheme, based on the idea of distributed lags, has been analyzed and estimated by Ghysels, Santa-Clara and Valkanov (2002).

temporaneous effect on returns, and similarly returns have a contemporaneous effect on volatility. These results are confirmed with both realized and bipower variations. Furthermore, dependence between volatility and returns is also economically and statistically important for several days.

Let us now consider a trivariate autoregressive model including implied volatility in addition to realized volatility (bipower variation) and returns, as suggested in Section 4 [figures 5 - 7 in Appendix B and figures 13 - 16 in Appendix C]. First, we see that implied volatility (IV) helps to predict future realized volatility for several days ahead [Figure 5 in Appendix B; Figure 13 in Appendix C]. It is also interesting to note that the difference between IV and RV, which captures a variance risk premium, also helps predict future volatility. Note that Bollerslev et al. (2006) do not consider implied volatility in their analysis.

Second, there is an important increase in the volatility feedback effect when implied volatility is taken into account [Figure 6 in Appendix B; figures 14 - 15 in Appendix C]. In particular, it is statistically significant during the first four days. The volatility feedback effect relies first on the volatility clustering phenomena which means that returns shocks, positive or negative, increases both current and future volatility. The second basic explanation of this hypothesis is that there is a positive intertemporal relationship between conditional volatility and expected returns. Thus, given the anticipative role of implied volatility and the link between the volatility feedback effect and future volatility, implied volatility reinforces and increases the impact of volatility on returns.[8] Figure 6 also compares volatility feedback effects with and without implied volatility as an auxiliary variable. We see that the difference between IV and RV has a stronger impact on returns than realized volatility alone in the presence of implied volatility. Further, different transformations of volatility (logarithmic of volatility and standard deviation) are considered: the volatility feedback effect is strongest when the standard deviation is used to measure volatility.

Finally, we look at the leverage effects with and without implied volatility as an auxiliary variable [Figure 16 in Appendix C]. We see that there is almost no change in the leverage effect when we take into account implied volatility. On comparing the leverage and volatility feedback effects with and without implied volatility, we see that the difference, in terms of causality measure, between leverage and volatility feedback effects decreases when implied volatility is included in the information set. In other words, taking into account implied volatility allows to identify a volatility feedback effect without affecting the leverage effect. This may reflect the fact that investors use several markets to carry out their financial strategies, and information is disseminated across several

[8] Since option prices reflect market participants' expectations of future movements of the underlying asset, the volatility implied from option prices should be an efficient forecast of future volatility, which potentially explains a better identification of the volatility feedback effect.

markets. Since the identification of a causal relationship depends crucially on the specification of the information set, including implied volatility appears essential to demonstrate a volatility feedback effect.

6. Dynamic impact of positive and negative news on volatility

In the previous sections, we did not account for the fact that return news may differently affect volatility depending on whether they are good or bad. We will now propose a method to sort out the differential effects of good and bad news, along with a simulation study showing that our approach can indeed detect asymmetric responses of volatility to return shocks.

6.1. Theory

Several volatility models capture this asymmetry and are explored in Engle and Ng (1993). To study the effect of current return shocks on future expected volatility, Engle and Ng (1993) introduced the News Impact Function (hereafter *NIF*). The basic idea of this function is to consider the effect of the return shock at time t on volatility at time $t+1$ in isolation while conditioning on information available at time t and earlier. Recently, Chen and Ghysels (2007) have extended the concept of news impact curves to the high-frequency data setting. Instead of taking a single horizon fixed parametric framework they adopt a flexible multi-horizon semi-parametric modeling [see also Linton and Mammen (2005)].

In what follows we extend our previous VAR model to capture the *dynamic* impact of bad news (negative innovations in returns) and good news (positive innovations in returns) on volatility. We quantify and compare the strength of these effects in order to determine the most important ones. To analyze the impact of news on volatility, we consider the following model:

$$\ln(\sigma^2_{t+1}) = \mu_\sigma + \sum_{j=1}^{p} \varphi_j^\sigma \ln(\sigma^2_{t+1-j}) + \sum_{j=1}^{p} \varphi_j^- \, er^-_{t+1-j} + \sum_{j=1}^{p} \varphi_j^+ \, er^+_{t+1-j} + u^\sigma_{t+1} \tag{6.1}$$

where

$$er^-_{t+1-j} = \min\left\{er_{t+1-j},\, 0\right\}, \;\; er^+_{t+1-j} = \max\left\{er_{t+1-j},\, 0\right\}, \;\; er_{t+1-j} = r_{t+1-j} - \mathsf{E}_{t-j}(r_{t+1-j}), \tag{6.2}$$

$$\mathsf{E}\left[u^\sigma_t\right] = 0 \text{ and } \mathsf{E}\left[u^\sigma_t u^\sigma_s\right] = \begin{cases} \Sigma_{u^\sigma} & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases}. \tag{6.3}$$

Equation (6.1) represents the linear projection of volatility on its own past and the past of centered negative and positive returns. This regression model allows one to capture the effect of centered negative or positive returns on volatility through the coefficients φ_j^- or φ_j^+ respectively, for $j = 1, \ldots, p$. It also allows one to examine the different effects that large and small negative and/or positive information shocks have on volatility. This will provide a check on the results obtained in the literature on GARCH modeling, which has put forward overwhelming evidence on the effect of negative shocks on volatility.

Again, in our empirical applications, σ_{t+1}^2 will be replaced by realized volatility RV_{t+1} or bipower variation BV_{t+1}. Furthermore, the conditional mean return will be approximated by the following rolling-sample average:

$$\hat{E}_t(r_{t+1}) = \frac{1}{m}\sum_{j=1}^{m} r_{t+1-j}.$$

where we take an average around $m = 15,\ 30,\ 90,\ 120,$ and 240 days. Now, let us consider the following restricted models:

$$\ln(\sigma_{t+1}^2) = \theta_\sigma + \sum_{i=1}^{\bar{p}} \bar{\varphi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\bar{p}} \bar{\varphi}_i^+ \, er_{t+1-j}^+ + e_{t+1}^\sigma, \tag{6.4}$$

$$\ln(\sigma_{t+1}^2) = \bar{\theta}_\sigma + \sum_{i=1}^{\dot{p}} \dot{\varphi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\dot{p}} \dot{\varphi}_i^- \, er_{t+1-j}^- + v_{t+1}^\sigma. \tag{6.5}$$

Equation (6.4) represents the linear projection of volatility $\ln(\sigma_{t+1}^2)$ on its own past and the past of centred positive returns. Similarly, equation (6.5) represents the linear projection of volatility $\ln(\sigma_{t+1}^2)$ on its own past and the past of centred negative returns. To compare the forecast error variances of model (6.1) with those of models (6.4) and (6.5), we assume that $p = \bar{p} = \dot{p}$.

In our empirical application, we also consider a model with uncentered negative and positive returns:

$$\ln(\sigma_{t+1}^2) = \omega_\sigma + \sum_{j=1}^{p} \phi_j^\sigma \ln(\sigma_{t+1-j}^2) + \sum_{j=1}^{p} \phi_j^- \, r_{t+1-j}^- + \sum_{j=1}^{p} \phi_j^+ \, r_{t+1-j}^+ + \epsilon_{t+1}^\sigma \tag{6.6}$$

where $r_{t+1-j}^- = \min\{r_{t+1-j}\,, 0\}\,,\ r_{t+1-j}^+ = \max\{r_{t+1-j}, 0\}\,,$

$$\mathsf{E}\left[\epsilon_t^\sigma\right] = 0 \text{ and } \mathsf{E}\left[\epsilon_t^\sigma \epsilon_s^\sigma\right] = \begin{cases} \Sigma_{\epsilon^\sigma} & \text{for } s = t \\ 0 & \text{for } s \neq t \end{cases}, \tag{6.7}$$

and the corresponding restricted volatility models:

$$\ln(\sigma_{t+1}^2) = \lambda_\sigma + \sum_{i=1}^{\bar{p}} \bar{\phi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\bar{p}} \bar{\phi}_i^+ \, r_{t+1-i}^+ + \upsilon_{t+1}^\sigma, \tag{6.8}$$

$$\ln(\sigma_{t+1}^2) = \bar{\lambda}_\sigma + \sum_{i=1}^{\dot{p}} \dot{\phi}_i^\sigma \ln(\sigma_{t+1-i}^2) + \sum_{i=1}^{\dot{p}} \dot{\phi}_i^- \, r_{t+1-i}^- + \varepsilon_{t+1}^\sigma. \tag{6.9}$$

Thus, a measure of the impact of bad news on volatility at horizon h, where $h \geq 1,$ is given by the following equation:

$$C(er^- \underset{h}{\rightarrow} \ln(\sigma^2)) = \ln\left[\frac{\mathsf{Var}\left[e_{t+h}^\sigma \mid \sigma^2(\omega,\, t],\;\; er^+(\omega,\, t]\right]}{\mathsf{Var}\left[u_{t+h}^\sigma \mid J(t)\right]}\right]. \tag{6.10}$$

Similarly, a measure of the impact of good news on volatility at horizon h is given by:

$$C(er^+ \underset{h}{\rightarrow} \ln(\sigma^2)) = \ln\left[\frac{\mathsf{Var}\left[v_{t+h}^\sigma \mid \sigma^2(\omega,\, t],\;\; er^-(\omega,\, t]\right]}{\mathsf{Var}\left[u_{t+h}^\sigma \mid J(t)\right]}\right] \tag{6.11}$$

where

$$er^-(\omega,\, t] \;\;=\;\; \left\{er_{t-s}^-,\;\; s \geq 0\right\}, \tag{6.12}$$

$$er^+(\omega,\, t] \;\;=\;\; \left\{er_{t-s}^+,\;\; s \geq 0\right\}, \tag{6.13}$$

and $J(t)$ is the information set obtained by "adding" $\sigma^2(\omega, t]$ to $er^-(\omega, t]$ and $er^+(\omega, t]$, introduced the News Impact Function (hereafter *NIF*). By analogy, e call the curves defined in (6.10) and (6.11), *Causal News Impact Functions* (*CNIF*). We also define a function which allows us to compare the impact of bad and good news on volatility. This function can be defined as follows:

$$C(er^- \mid er^+ \underset{h}{\rightarrow} \ln(\sigma^2)) = \ln\left[\frac{\mathsf{Var}\left[e_{t+h}^\sigma \mid \sigma^2(\omega,\, t],\;\; er^+(\omega,\, t]\right]}{\mathsf{Var}\left[v_{t+h}^\sigma \mid \sigma^2(\omega,\, t],\;\; er^-(\omega,\, t]\right]}\right]. \tag{6.14}$$

When $C(er^- \mid er^+ \underset{h}{\rightarrow} \ln(\sigma^2)) \geq 0,$ this means that bad news have more impact on volatility than good news. Otherwise, good news will have more impact on volatility than bad news. Compared to Chen and Ghysels (2007), our approach is also multi-horizon and based on high-frequency data but is more parametric in nature. Before applying these new measures to our S&P 500 futures market, we conduct a simulation study to verify that the asymmetric reaction of volatility is well captured in various models of the GARCH family that produce or not such an asymmetry.

6.2. Simulation study on news asymmetry detection

We will now present an exploratory simulation study on the ability of the causality measures to detect asymmetry in the impact of bad and good news on volatility [Pagan and Schwert (1990), Gouriéroux and Monfort (1992), Engle and Ng (1993)]. To do this, we consider that returns are governed by a process of the form:

$$r_{t+1} = \sqrt{\sigma_t}\varepsilon_{t+1} \tag{6.15}$$

where $\varepsilon_{t+1} \sim \mathcal{N}(0, 1)$ and σ_t represents the conditional volatility of return r_{t+1}. Since we are only interested in studying the asymmetry in leverage effect, equation (6.15) does not allow for a volatility feedback effect. Second, we assume that σ_t follows one of the following heteroskedastic models:

1. GARCH$(1,1)$ model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\varepsilon_{t-1}^2\,; \tag{6.16}$$

2. EGARCH$(1,1)$ model:

$$\log(\sigma_t) = \omega + \beta\log(\sigma_{t-1}) + \gamma\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} + \alpha\left[\frac{\mid \varepsilon_{t-1} \mid}{\sqrt{\sigma_{t-1}}} - \sqrt{2/\pi}\right]\;; \tag{6.17}$$

3. nonlinear NL-GARCH$(1,1)$ model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha \mid \varepsilon_{t-1} \mid^{\gamma}; \tag{6.18}$$

4. GJR-GARCH$(1,1)$ model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 \tag{6.19}$$

where

$$I_{t-1} = \begin{cases} 1, & \text{if } \varepsilon_{t-1} \leq 0\,, \\ 0, & \text{otherwise;} \end{cases}$$

5. asymmetric AGARCH$(1,1)$ model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\left(\varepsilon_{t-1} + \gamma\right)^2\;; \tag{6.20}$$

6. VGARCH$(1,1)$ model:

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\left(\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}}} + \gamma\right)^2\;; \tag{6.21}$$

7. nonlinear asymmetric GARCH$(1,1)$ model [NGARCH$(1,1)$] :

$$\sigma_t = \omega + \beta\sigma_{t-1} + \alpha\left(\varepsilon_{t-1} + \gamma\sqrt{\sigma_{t-1}}\right)^2 . \tag{6.22}$$

GARCH and NL-GARCH models are, by construction, symmetric. Thus, we expect that the curves of causality measures for bad and good news will be the same. Similarly, because EGARCH, GJR-GARCH, AGARCH, VGARCH, and NGARCH are asymmetric we expect that these curves will be different. The parameter values considered are given in Table 4 of Appendix B.[9]

To see whether the asymmetric structures gets translated into the causality patterns, we then simulate returns and volatilities according to the above models and we evaluate the causality measures for bad and good news as described in Section 6.1. To abstract from statistical uncertainty, the models are simulated with a large sample size ($T = 40000$).

The results obtained in this way are reported in Figure 8 of Appendix B. We see from these that symmetry and asymmetry are well represented by causality measure patterns. For the symmetric models [GARCH and NL-GARCH], bad and good news have the same impact on volatility. In contrast, for the asymmetric models [EGARCH, GJR-GARCH, AGARCH, VGARCH, NGARCH], bad and good news exhibit different impact curves.

It is also interesting to observe for the asymmetric models that bad news have a greater impact on volatility than good news. The magnitude of the volatility response is largest for NGARCH model, followed by the AGARCH and GJR-GARCH models. The effect is negligible in EGARCH and VGARCH models. The impact of good news on volatility is more noticeable in AGARCH and NGARCH models. Overall, causality measures appear to capture quite well the effects of returns on volatility, both qualitatively and quantitatively.

7. News effects in S&P 500 futures market

We now apply the good news and bad news measures of causality to S&P 500 futures returns. To carry out our analysis, we consider two alternative measures of news: (1) positive and negative deviations of returns from average past returns, and (2) positive and negative variance risk premia. An important feature of our approach comes from the fact that a specific volatility model need not be estimated, which can be contrasted with previous related studies [see, for example Bekaert and Wu (2000), Engle and Ng (1993), Glosten et al. (1993), Campbell and Hentschel (1992), and Nelson

[9]These parameters are the results of an estimation of different parametric volatility models using the daily returns series of the Japanese TOPIX index from January 1, 1980 to December 31, 1988. For details, see Engle and Ng (1993). We also considered other values based on Engle and Ng (1993). The results are similar to those presented here.

(1991)].

7.1. Return news

Our empirical results on return news effect (including *Causal News Impact Functions*) are summarized and compared in Figure 9 of Appendix B. Detailed results (with confidence intervals) are presented in tables 5-7 of Appendix B and figures 17-C. We find a much stronger impact of bad news on volatility for several days. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons. So our central finding is that bad news have more impact on volatility than good news at all horizons.

7.2. Variance risk premium

Let us now look at the reaction of future returns to the sign of the difference between implied volatility and realized volatility (bipower variation). This difference is a measure of the variance risk premium since the option-implied volatility includes the risk premium that investors associate with expected future volatility [see Bollerslev and Zhou (2006) and Drechsler and Yaron (2008)]. We will therefore assess whether a positive variance risk premium has an impact of similar magnitude on expected returns than a negative variance risk premium. In the case of a positive variance risk premium, we expect an increase in the expected returns (return risk premium), and in the opposite, we expect a decrease in expected returns.

Since implied volatility is a predictor of future volatility, we write:

$$\ln(RV_{t+h}) = f\left(\ln(IV_t),\ \ \ln(IV_{t-1}),\, \ldots\right) + \varepsilon_{t+h},\ \ \forall h \geq 1, \tag{7.1}$$

$$\varepsilon_{t+h} = \ln(RV_{t+h}) - f\left(\ln(IV_t),\ \ \ln(IV_{t-1}),\, \ldots\right), \tag{7.2}$$

where $f\left(\ln(IV_t),\ \ln(IV_{t-1}),\, \ldots\right)$ is a function of the past observations on implied volatility.[10] The term on the right-hand side of equation (7.2) can be viewed as an approximation of volatility shocks or volatility news. To measure empirically the *dynamic* impact of volatility news on returns, we consider the following model:

$$r_{t+1} = \mu_r + \sum_{j=1}^{p} \varphi_j^r r_{t+1-j} + \sum_{j=1}^{p} \varphi_j^-\, VP^-_{t+1-j} + \sum_{j=1}^{p} \varphi_j^+\, VP^+_{t+1-j} + u^r_{t+1} \tag{7.3}$$

[10] $f\left(\ln(IV_t),\ \ln(IV_{t-1}), \ldots\right)$ represents the optimal forecast, in the sense of minimization of the mean squared error, of $\ln(RV_{t+h})$ based on the past observations of implied volatility.

where $VP^{-}_{t+1-j} = \min\{VP_{t+1-j},\, 0\}$, $VP^{+}_{t+1-j} = \max\{VP_{t+1-j},\, 0\}$ and

$$VP_{t+1-j} = \ln(IV_{t+1-j}) - \ln(RV_{t+1-j})\,, \quad j = 1,\, \ldots\,,\, p. \tag{7.4}$$

Equation (7.3) represents a linear projection of returns on its own past and the past of negative and positive variance risk premia. This regression model allows one to capture the effect of volatility news on returns through the coefficients φ^{-}_{j} or φ^{+}_{j}, for $j = 1, \ldots, p$. It also allows one to examine different effects that large and small negative and/or positive volatility shocks have on return risk premium. When implied volatility is bigger than realized volatility (bipower variation), we expect an increase in future volatility followed by an increase in the expected returns. In the opposite situation, we expect a decrease in future volatility followed by a decrease in the expected returns.

The empirical results on the impact of volatility news on returns are given in Figure 10 of Appendix B. The latter shows the impact of negative and positive variance risk premium on returns and the comparison between them. We see that a positive variance risk premium has more impact on expected returns than a negative variance risk premium, which means that positive shocks on volatility have more impact on returns than negative shocks. The impact is twice as big on the first day and shrinks to zero after about five days. By looking at the sign of coefficients φ^{+}_{j} and φ^{-}_{j}, for $j = 1, \ldots, p$, we find that φ^{+}_{j} are positive, whereas φ^{-}_{j} are negative, as expected. Consequently, the increase in expected returns tends to be higher than the decrease for a movement in the variance risk premium of the same magnitude but of opposite signs.

8. Conclusion

In this paper we analyze and quantify the relationship between volatility and returns with high-frequency equity returns. Within the framework of a vector autoregressive linear model of returns and realized volatility or bipower variation, we quantify the dynamic leverage and volatility feedback effects by applying short-run and long-run causality measures proposed by Dufour and Taamouti (2009). These causality measures go beyond simple correlation measures used recently by Bollerslev et al. (2006).

Using 5-minute observations on S&P 500 Index futures contracts, we measure a weak dynamic leverage effect for the first four hours in hourly data and a strong dynamic leverage effect for the first three days in daily data. The volatility feedback effect is found to be negligible at all horizons. Interestingly, when we remeasure the dynamic leverage and volatility feedback effects using implied volatility (IV), we find that a volatility feedback effect appears, while the leverage effect remains almost the same. This can be explained by the power of implied volatility to predict future volatility and by the fact that volatility feedback effect is related to the latter. We also use causality measures to quantify and test statistically the dynamic impact of good and bad news on volatility. First, we assess by simulation the ability of causality measures to detect the differential effect of good and bad news in various parametric volatility models. Then, empirically, we measure a much stronger impact for bad news at several horizons. Statistically, the impact of bad news is significant for the first four days, whereas the impact of good news is negligible at all horizons. We introduce a new concept of news based on volatility. This one is defined by the difference between implied volatility and realized volatility (bipower variation). When implied volatility is bigger than realized volatility (bipower variation) it means that the market is expecting an increase in future volatility with respect to current volatility. Our empirical results show that such an expected increase in volatility has a stronger impact on return risk premium than an expected decrease of a similar magnitude.

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Appendix

A. Bootstrap confidence intervals for causality measures

We compute the nominal 95% bootstrap confidence intervals of the causality measures as follows [see Dufour and Taamouti (2009)]:

(1) Estimate by OLS the $VAR(p)$ process given by equation (3.1) and save the residuals[11]

$$\hat{u}(t) = \begin{pmatrix} r_t \\ \ln(RV_t) \end{pmatrix} - \hat{\mu} - \sum_{j=1}^{p} \hat{\Phi}_j \begin{pmatrix} r_{t-j} \\ \ln(RV_{t-j}) \end{pmatrix}, \text{ for } t = p+1, \ldots, T, \tag{A.1}$$

where $\hat{\mu}$ and $\hat{\Phi}_j$ are the OLS regression estimates of μ and Φ_j, for $j = 1, \ldots, p$.

(2) Generate $(T$-$p)$ bootstrap residuals $\hat{u}^*(t)$ by random sampling with replacement from the residuals $\hat{u}(t),\ t = p+1, \ldots, T$.

(3) Generate a random draw for the vector of p initial observations

$$w(0) = [(r_1, \ln((RV_1))', \ldots, (r_p, \ln(RV_p))']'. \tag{A.2}$$

(4) Given $\hat{\mu}$ and $\hat{\Phi}_j,$ for $j = 1, \ldots, p,\ \hat{u}^*(t),$ and $w(0),$ generate bootstrap data for the dependent variable $(r_t^*, \ln(RV_t)^*)'$ from equation:

$$\begin{pmatrix} r_t^* \\ \ln(RV_t)^* \end{pmatrix} = \hat{\mu} + \sum_{j=1}^{p} \hat{\Phi}_j \begin{pmatrix} r_{t-j}^* \\ \ln(RV_{t-j})^* \end{pmatrix} + \hat{u}^*(t), \quad t = p+1, \ldots, T. \tag{A.3}$$

(5) Calculate the bootstrap OLS regression estimates

$$\hat{\Phi}^* = (\hat{\mu}^*, \hat{\Phi}_1^*, \hat{\Phi}_2^*, \ldots, \hat{\Phi}_p^*) = \hat{\Gamma}^{*-1}\hat{\Gamma}_1^*, \ \hat{\Sigma}_u^* = \sum_{t=p+1}^{T} \hat{u}^*(t)\hat{u}^*(t)'/(T-p), \tag{A.4}$$

$$\hat{\Gamma}^* = (T-p)^{-1} \sum_{t=p+1}^{T} w^*(t)w^*(t)', \ \hat{\Gamma}_1^* = (T-p)^{-1} \sum_{t=p+1}^{T} w^*(t)(r_{t+1}^*, \ln(RV_{t+1})^*)' \tag{A.5}$$

[11] When we "add" the past of implied volatility to the information set $I_{r\sigma^2}(t)$, then we consider the $VAR(p)$ process given by equation (4.1).

where $w^*(t) = [(r_t^*, \ln(RV_t)^*)', \ldots, (r_{t-p+1}^*, \ln(RV_{t-p+1})^*)']'$ and

$$\hat{u}^*(t) = \tilde{u}^*(t) - \sum_{t=p+1}^{T} \tilde{u}^*(t)/(T-p), \text{ and } \tilde{u}^*(t) = \begin{pmatrix} r_t^* \\ \ln(RV_t)^* \end{pmatrix} - \hat{\mu} - \sum_{j=1}^{p} \hat{\varPhi}_j \begin{pmatrix} r_{t-j}^* \\ \ln(RV_{t-j})^* \end{pmatrix}. \tag{A.6}$$

(6) Estimate the constrained model of the marginal process r_t and $\ln(RV_t)$ using the bootstrap sample $\left\{(r_t^*, \ln(RV_t))'\right\}_{t=1}^{T}$.
(7) Calculate the causality measures at horizon h, denoted $\hat{C}^{(j)*}(r \underset{h}{\longrightarrow} \ln(RV))$ and $\hat{C}^{(j)*}(\ln(RV) \underset{h}{\longrightarrow} r)$, using equations (3.17) and (3.18) respectively and the bootstrap sample.
(8) Choose B such $\frac{1}{2}\alpha(B+1)$ is an integer and repeat steps (2)-(7) B times.[12]
(9) Finally, calculate the α and 1-α percentile interval endpoints of the distributions of $\hat{C}^{(j)*}(r \underset{h}{\longrightarrow} \ln(RV))$ and $\hat{C}^{(j)*}(\ln(RV) \underset{h}{\longrightarrow} r)$.[13]
A proof of the asymptotic validity of the bootstrap confidence intervals of the causality measures is provided in Dufour and Taamouti (2009).

[12] 1-α is the considered level of confidence interval.
[13] We follow the same steps to compute the bootstrap confidence intervals of instantaneous causality and dependence measures.

B. Summary of empirical results

We present here basic summary statistics and graphs for the data used in this paper.

Table 1. Summary statistics for S&P 500 futures returns,1988-2005

$Variables$	$Mean$	$St.Dev.$	$Median$	$Skewness$	$Kurtosis$
$Five$-minute	$6.9505e-006$	0.000978	$0.00e-007$	-0.0818	73.9998
$Hourly$	$1.3176e-005$	0.0031	$0.00e-007$	-0.4559	16.6031
$Daily$	$1.4668e-004$	0.0089	$1.1126e-004$	-0.1628	12.3714

Note: This table summarizes the five-minute, hourly, and daily returns distributions for the S&P 500 index contracts. The sample covers the period from 1988 to December 2005 for a total of 4494 trading days.

Table 2. Summary statistics for daily volatilities, 1988-2005

$Variables$	$Mean$	$St.Dev.$	$Median$	$Skewness$	$Kurtosis$
RV_t	$8.1354e-005$	$1.2032e-004$	$4.9797e-005$	8.1881	120.7530
BV_t	$7.6250e-005$	$1.0957e-004$	$4.6956e-005$	6.8789	78.9491
$\ln(RV_t)$	-9.8582	0.8762	-9.9076	0.4250	3.3382
$\ln(BV_t)$	-9.9275	0.8839	-9.9663	0.4151	3.2841
J_{t+1}	0.0870	0.1005	0.0575	1.6630	7.3867

Note: This table summarizes the daily volatilities distributions for the S&P 500 index contracts. The sample covers the period from 1988 to December 2005 for a total of 4494 trading days.

Table 3. Hourly and daily volatility feedback effects

Hourly volatility feedback effects using $\ln(RV)$

$C(\ln(RV) \underset{h}{\rightarrow} r)$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.00016	0.00014	0.00012	0.00012
95% Bootstrap interval	$[0.0000,\ 0.0007]$	$[0.0000,\ 0.0006]$	$[0.0000,\ 0.0005]$	$[0.0000,\ 0.0005]$

Hourly volatility feedback effects using $\ln(BV)$

$C(\ln(BV) \underset{h}{\rightarrow} r)$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.00022	0.00020	0.00019	0.00015
95% Bootstrap interval	$[0.0000,\ 0.0008]$	$[0.0000,\ 0.0007]$	$[0.0000,\ 0.0007]$	$[0.0000, 0.0005]$

Daily volatility feedback effects using $\ln(RV)$

$C(\ln(RV) \underset{h}{\rightarrow} r)$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0019	0.0019	0.0019	0.0011
95% Bootstrap interval	$[0.0007,\ 0.0068]$	$[0.0005,\ 0.0065]$	$[0.0004,\ 0.0061]$	$[0.0002,\ 0.0042]$

Daily volatility feedback effects using $\ln(BV)$

$C(\ln(BV) \underset{h}{\rightarrow} r)$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0017	0.0017	0.0016	0.0011
95% Bootstrap interval	$[0.0007,\ 0.0061]$	$[0.0005,\ 0.0056]$	$[0.0004,\ 0.0055]$	$[0.0002,\ 0.0042]$

Note: This table summarizes the estimation results of causality measures from hourly realized volatility $[\ln(RV)]$ to hourly returns (r), hourly bipower variation $[\ln(BV)]$ to hourly returns, daily realized volatility to daily returns, and daily bipower variation to daily returns, respectively. The second row in each small table gives the point estimate of the causality measures at horizons $h = 1, ..., 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Table 4. Parameter values of different GARCH models

	ω	β	α	γ
GARCH	2.7910^{-5}	0.86695	0.093928	$-$
EGARCH	-0.290306	0.97	0.093928	-0.09
NL-GARCH	2.7910^{-5}	0.86695	0.093928	$0.5,\ 1.5,\ 2.5$
GJR-GARCH	2.7910^{-5}	0.8805	0.032262	0.10542
AGARCH	2.7910^{-5}	0.86695	0.093928	-0.1108
VGARCH	2.7910^{-5}	0.86695	0.093928	-0.1108
NGARCH	2.7910^{-5}	0.86695	0.093928	-0.1108

Note: This table summarizes the parameter values for parametric volatility models considered in our simulations study.

Table 5. Measuring the impact of good news on volatility using $\ln(RV)$ [centered positive returns]

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{15}\sum_{j=1}^{15} r_{t+1-j}$$

$C(er^+ \underset{h}{\to} \ln(RV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0007	0.0007	0.0007	0.0004
95% Percentile bootstrap interval	[0.0003, 0.0043]	[0.0002, 0.0039]	[0.0001, 0.0034]	[0.0000, 0.0030]

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{30}\sum_{j=1}^{30} r_{t+1-j}$$

$C(er^+ \underset{h}{\to} \ln(RV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0010	0.0007	0.0007	0.0005
95% Percentile bootstrap interval	[0.0004, 0.0051]	[0.0003, 0.0039]	[0.0003, 0.0036]	[0.0000, 0.0032]

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{90}\sum_{j=1}^{90} r_{t+1-j}$$

$C(er^+ \underset{h}{\to} \ln(RV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0013	0.0008	0.0008	0.0008
95% Percentile bootstrap interval	[0.0004, 0.0059]	[0.0003, 0.0044]	[0.0002, 0.0041]	[0.0001, 0.0039]

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{120}\sum_{j=1}^{120} r_{t+1-j}$$

$C(er^+ \underset{h}{\to} \ln(RV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0011	0.00076	0.00072	0.00074
95% Percentile bootstrap interval	[0.0004, 0.0054]	[0.00029, 0.0041]	[0.00024, 0.00386]	[0.0000, 0.00388]

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{240}\sum_{j=1}^{240} r_{t+1-j}$$

$C(er^+ \underset{h}{\to} \ln(RV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0011	0.0006	0.0006	0.0007
95% Percentile bootstrap interval	[0.0004, 0.0053]	[0.0003, 0.0041]	[0.0002, 0.0035]	[0.0000, 0.0034]

Note: This table summarizes the estimation results of causality measures from centered positive returns (er^+) to realized volatility $[\ln(RV)]$ using five estimators of the conditional mean, for $m = 15,\ 30,\ 90,\ 120,\ 240$. In each of the five small tables, the second row gives the point estimate of the causality measures at horizons $h = 1, ..., 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Table 6. Measuring the impact of good news on volatility using $\ln(BV)$ [centered positive returns]

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{15}\sum_{j=1}^{15} r_{t+1-j}$$

$C(er^+ \underset{h}{\rightarrow} \ln(BV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0008	0.0008	0.0006	0.0006
95% Percentile bootstrap interval	$[0.0003, 0.0045]$	$[0.0002, 0.0041]$	$[0.0002, 0.0035]$	$[0.0000, 0.0034]$

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{30}\sum_{j=1}^{30} r_{t+1-j}$$

$C(er^+ \underset{h}{\rightarrow} \ln(BV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0012	0.0007	0.0007	0.0007
95% Percentile bootstrap interval	$[0.0005, 0.0053]$	$[0.0003, 0.0041]$	$[0.0002, 0.0039]$	$[0.0001, 0.0038]$

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{90}\sum_{j=1}^{90} r_{t+1-j}$$

$C(er^+ \underset{h}{\rightarrow} \ln(BV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0018	0.0009	0.0008	0.0010
95% Percentile bootstrap interval	$[0.0006, 0.0065]$	$[0.0003, 0.0044]$	$[0.0002, 0.0041]$	$[0.0001, 0.0042]$

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{120}\sum_{j=1}^{120} r_{t+1-j}$$

$C(er^+ \underset{h}{\rightarrow} \ln(BV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0016	0.0008	0.0007	0.0009
95% Percentile bootstrap interval	$[0.0006, 0.0063]$	$[0.0002, 0.0047]$	$[0.0002, 0.0042]$	$[0.0001, 0.0044]$

$$\widehat{\mathsf{E}_t(r_{t+1})} = \frac{1}{240}\sum_{j=1}^{240} r_{t+1-j}$$

$C(er^+ \underset{h}{\rightarrow} \ln(BV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0015	0.0007	0.0006	0.0008
95% Percentile bootstrap interval	$[0.0005, 0.0057]$	$[0.0002, 0.0044]$	$[0.0002, 0.0038]$	$[0.0001, 0.0037]$

Note: This table summarizes the estimation results of causality measures from centered positive returns (er^+) to bipower variation $[\ln(BV)]$ using five estimators of the conditional mean, for $m = 15,\ 30,\ 90,\ 120,\ 240$. In each of the five small tables, the second row gives the point estimate of the causality measures at horizons $h = 1, ..., 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Table 7. Measuring the impact of good news on volatility [uncentered positive returns]

using $\ln(RV)$

$C(r^{+} \underset{h}{\rightarrow} \ln(RV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0027	0.0012	0.0008	0.0009
95% Percentile bootstrap interval	$[0.0011,\ 0.0077]$	$[0.0004,\ 0.0048]$	$[0.0002,\ 0.0041]$	$[0.0001,\ 0.0038]$

using $\ln(BV)$

$C(r^{+} \underset{h}{\rightarrow} \ln(BV))$	$h=1$	$h=2$	$h=3$	$h=4$
Point estimate	0.0035	0.0013	0.0008	0.0010
95% Percentile bootstrap interval	$[0.0016,\ 0.0087]$	$[0.0004,\ 0.0051]$	$[0.0002,\ 0.0039]$	$[0.0001, 0.0043]$

Note: This table summarizes the estimation results of causality measures from uncentered positive returns (r^{+}) to realized volatility $[\ln(RV)]$ [bipower variation $\ln(BV)$]. The second row of each small table gives the point estimate of the causality measures at horizons $h = 1, \ldots, 4$. The third row gives the 95% corresponding percentile bootstrap interval.

Figure 1. Daily prices and returns of the S&P 500 futures. January 1988 to December 2005.

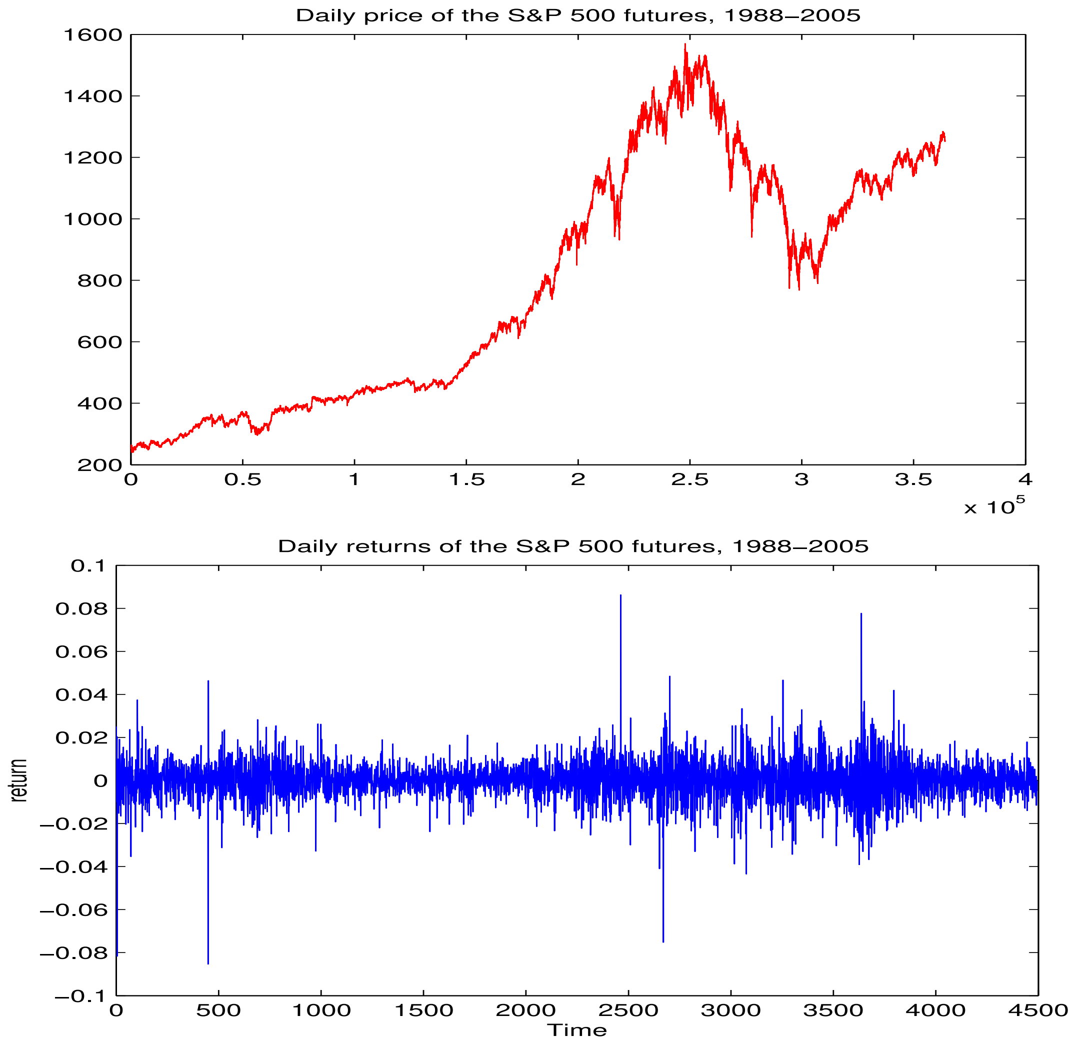


Figure 2. Daily realized volatility and bipower variation of the S&P 500 futures. January 1988 to December 2005.

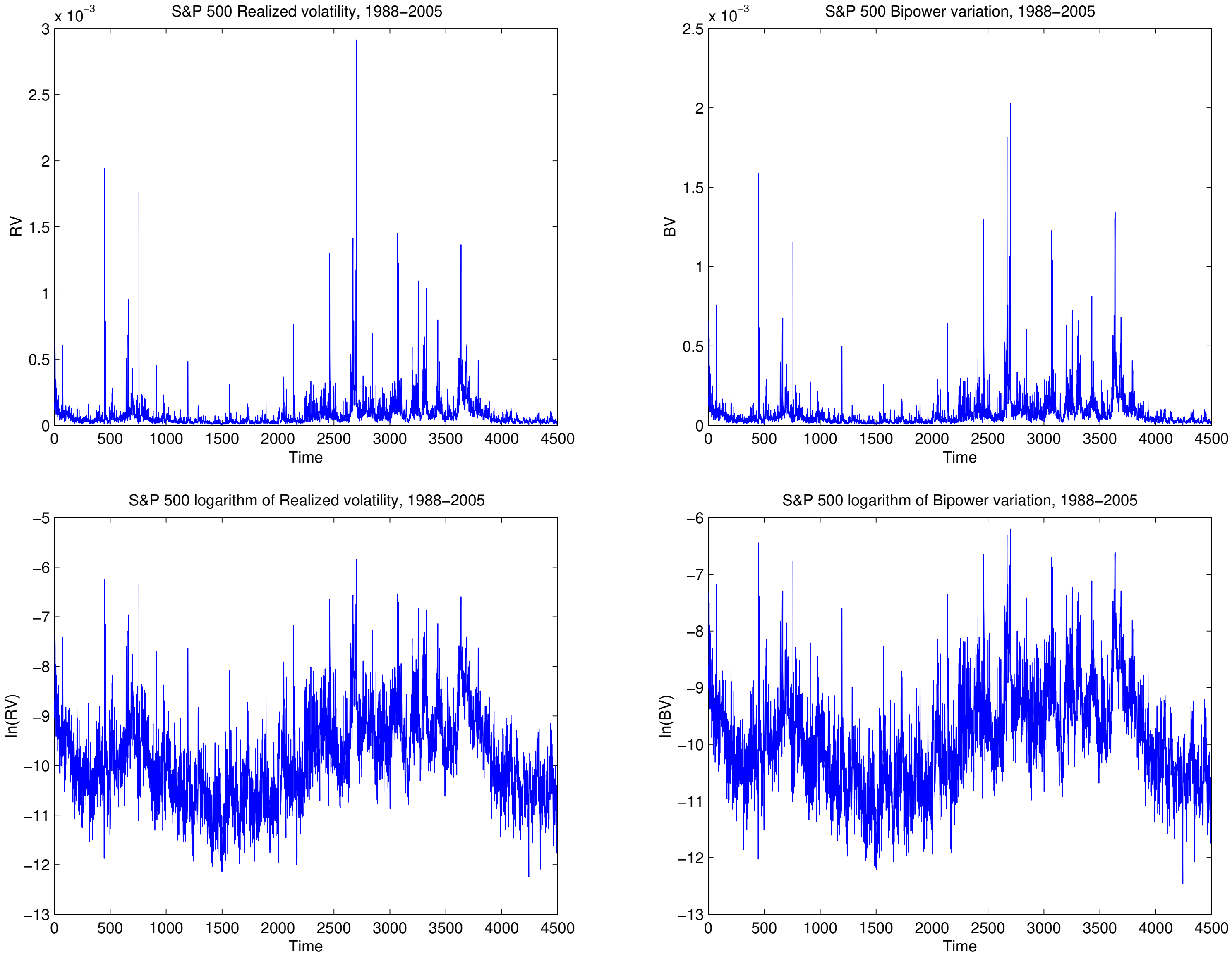


Figure 3. Quantile to quantile plots (QQ plot) of the relative measure of jumps (RJ), $z_{QP,l,t}$, $z_{QP,t}$, and $z_{QP,lm,t}$. January 1988 to December 2005.

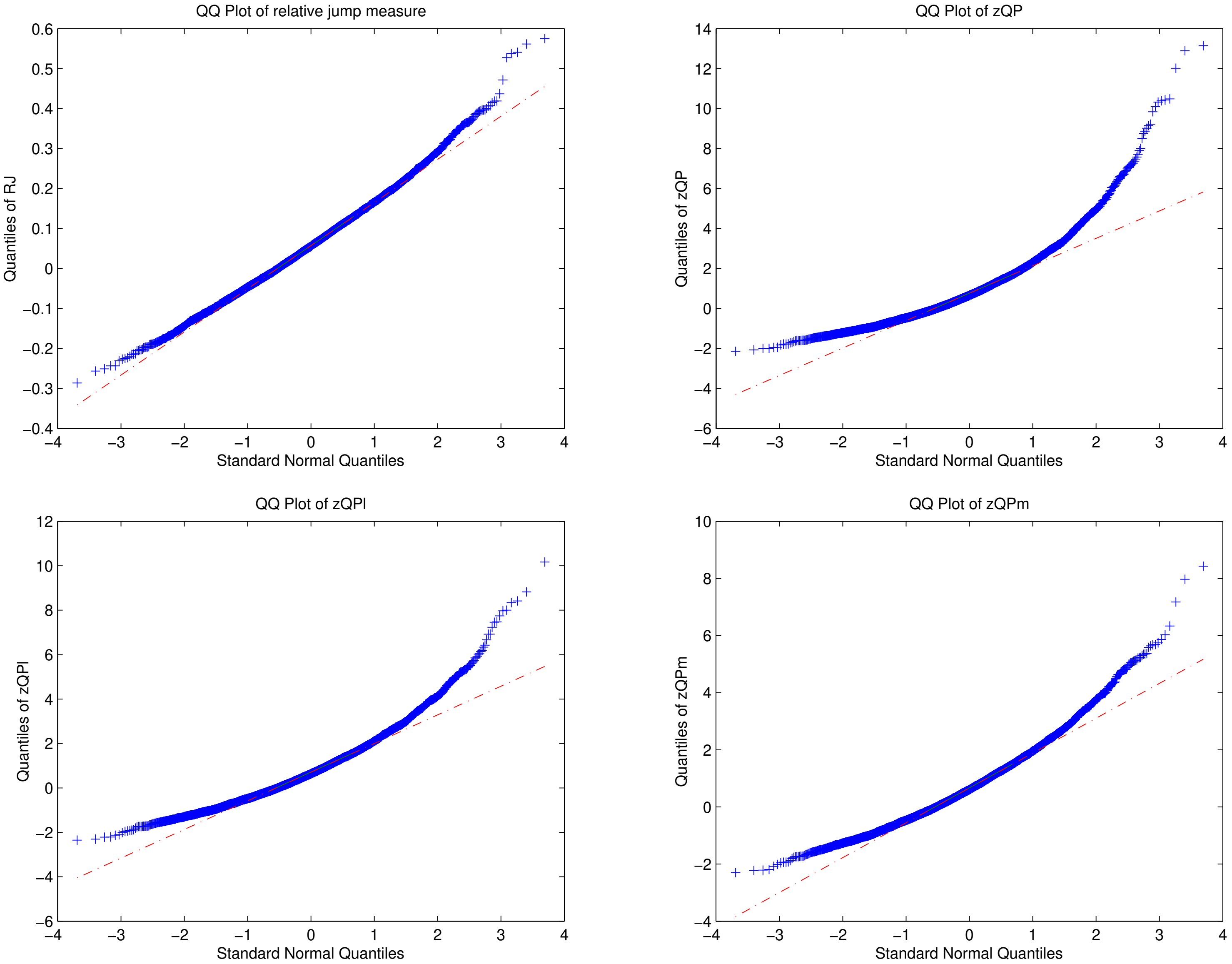


Figure 4. Leverage and volatility feedback effects in hourly and daily data using a bivariate autoregressive model (r, RV). January 1988 to December 2005.

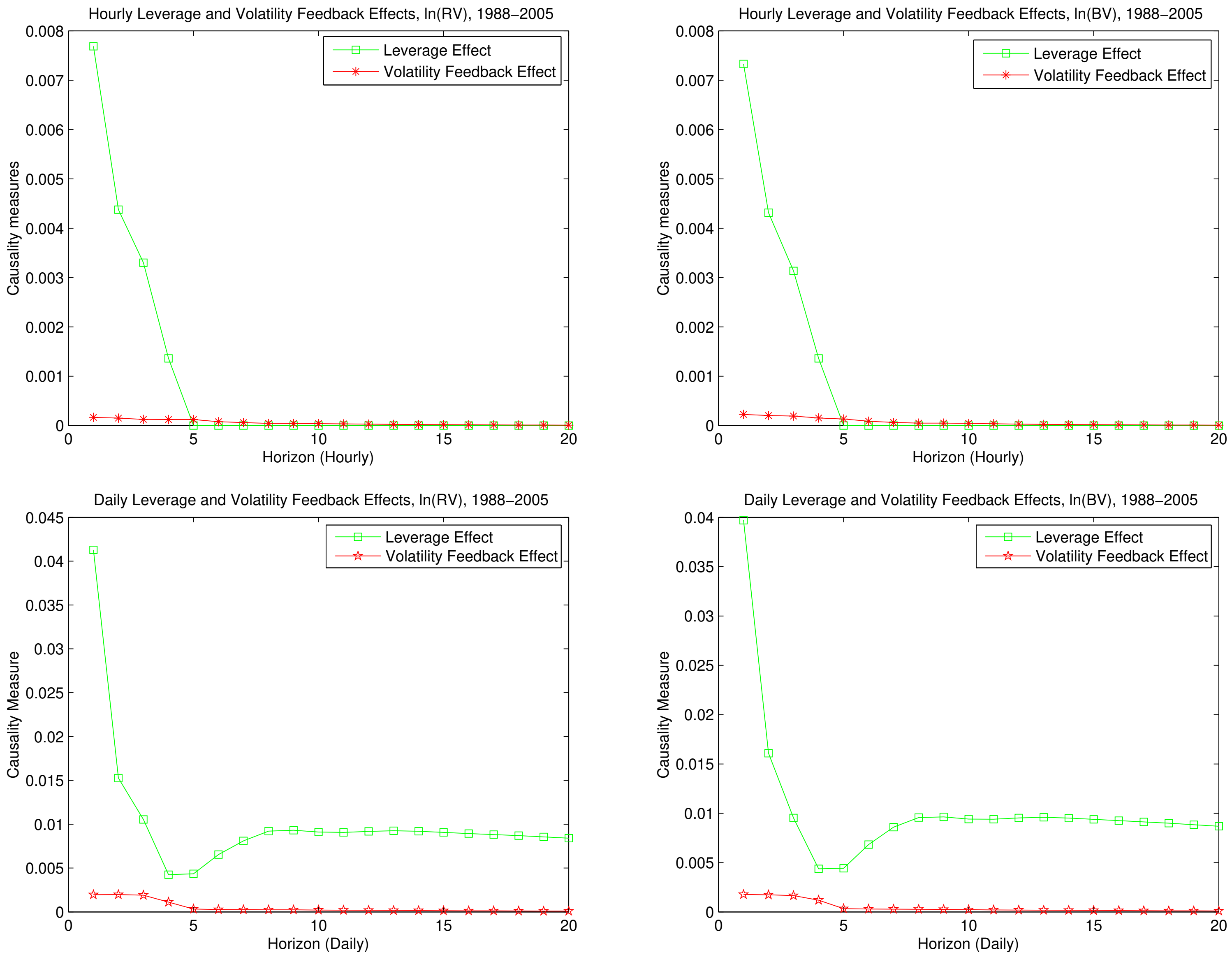


Figure 5. Causality measures between implied volatility (IV) [or variance risk premium $IV - RV$] and realized volatility (RV), using trivariate VAR models for (r, RV, IV) and $(r, RV, IV - RV)$. January 1996 to December 2005.

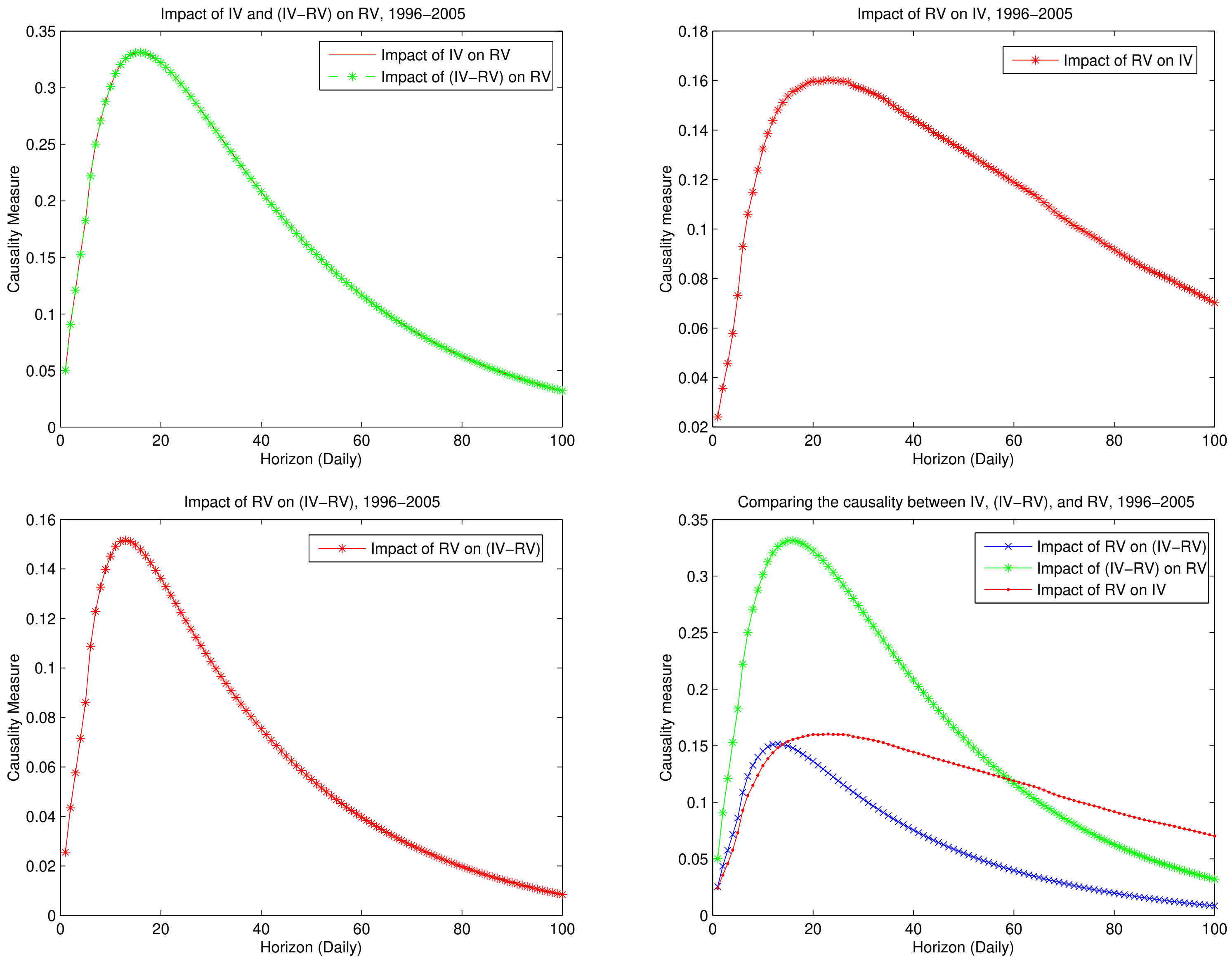


Figure 6. Volatility feedback effects, with implied volatility as auxiliary variable [trivariate models (r, RV, IV) and $(r, RV, IV - RV)$] and without implied volatility [bivariate model (r, RV)]; different transformations of volatility considered. Impact of vector $(RV,\ IV - RV)$ on returns. January 1996 to December 2005.

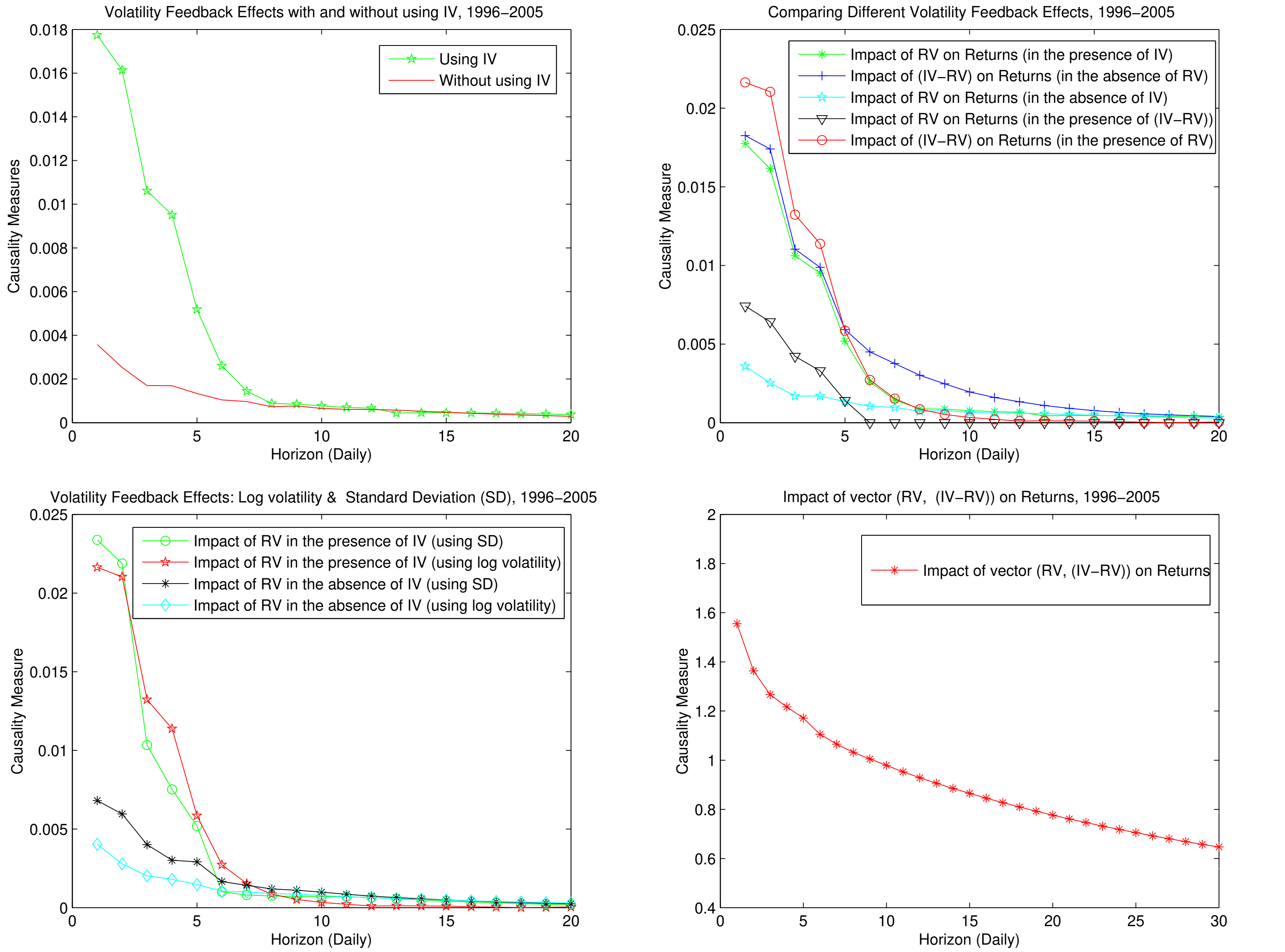


Figure 7. Leverage and volatility feedback effects, with implied volatility as auxiliary variable [trivariate models (r, RV, IV) and $(r, RV, IV - RV)$] and without implied volatility [bivariate model (r, RV)]. January 1996 to December 2005.

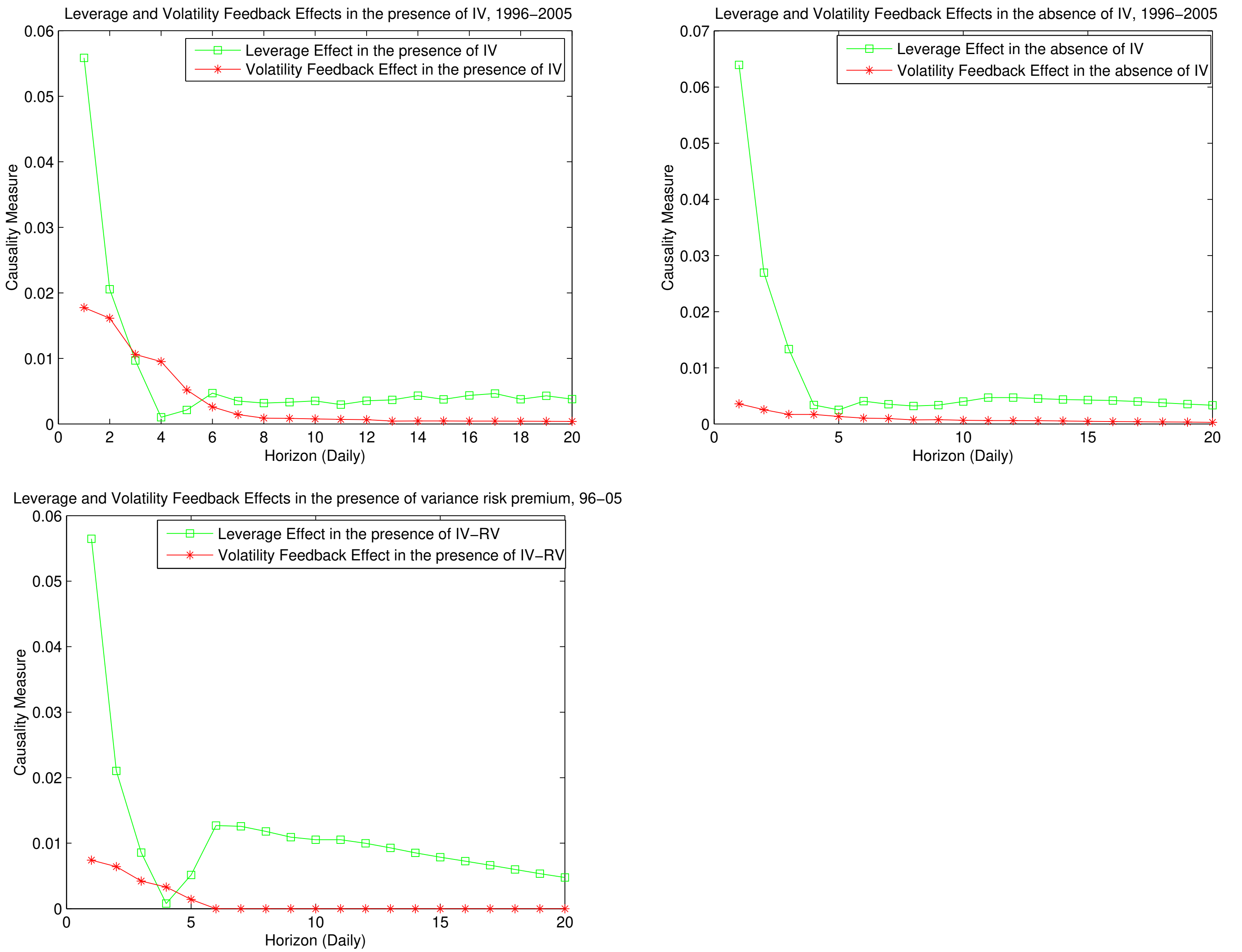


Figure 8. Causality measures of the impact of bad and good news on symmetric and asymmetric GARCH volatility models.

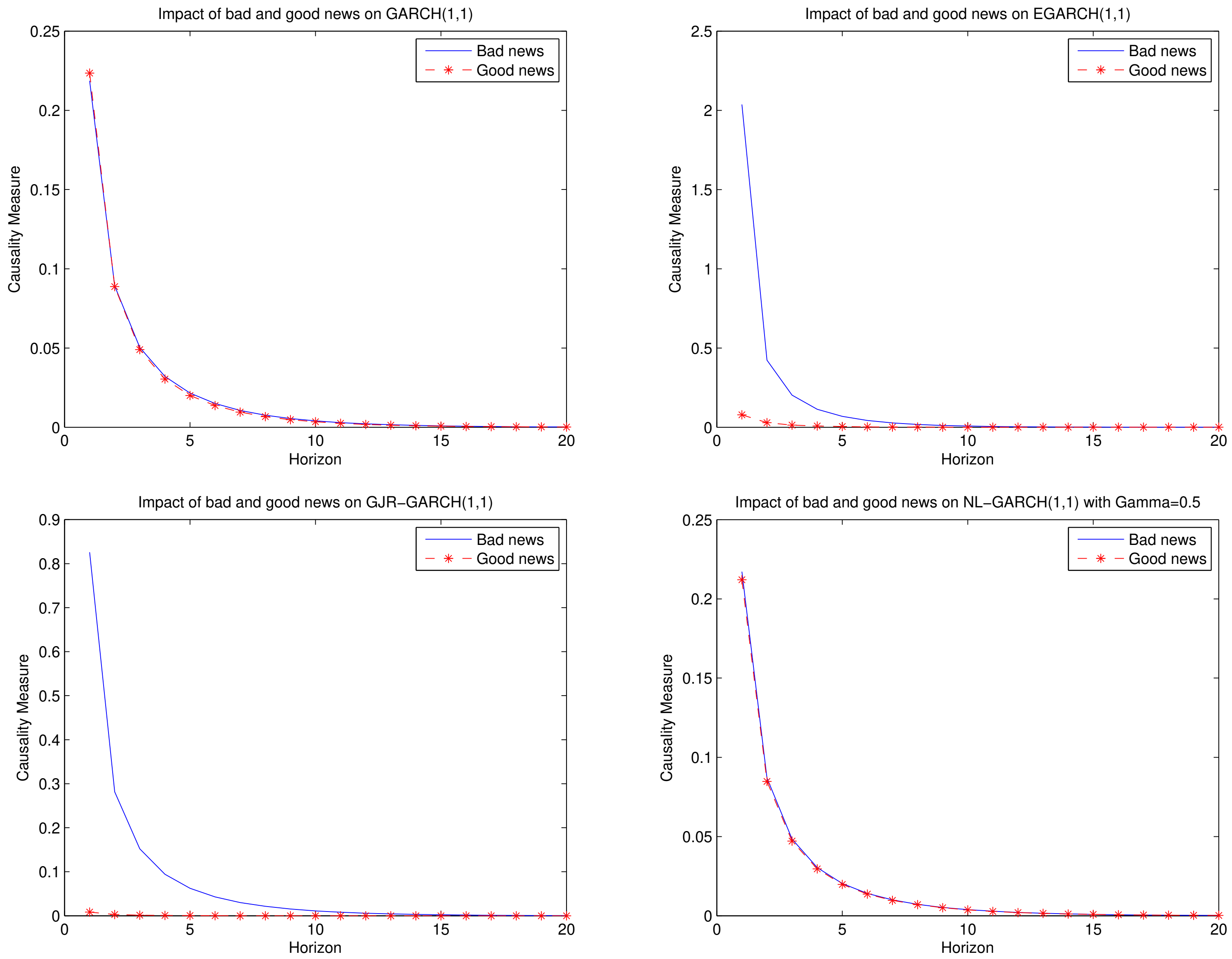


Figure 8 (continued). Causality measures of the impact of bad and good news on symmetric and asymmetric GARCH volatility models.

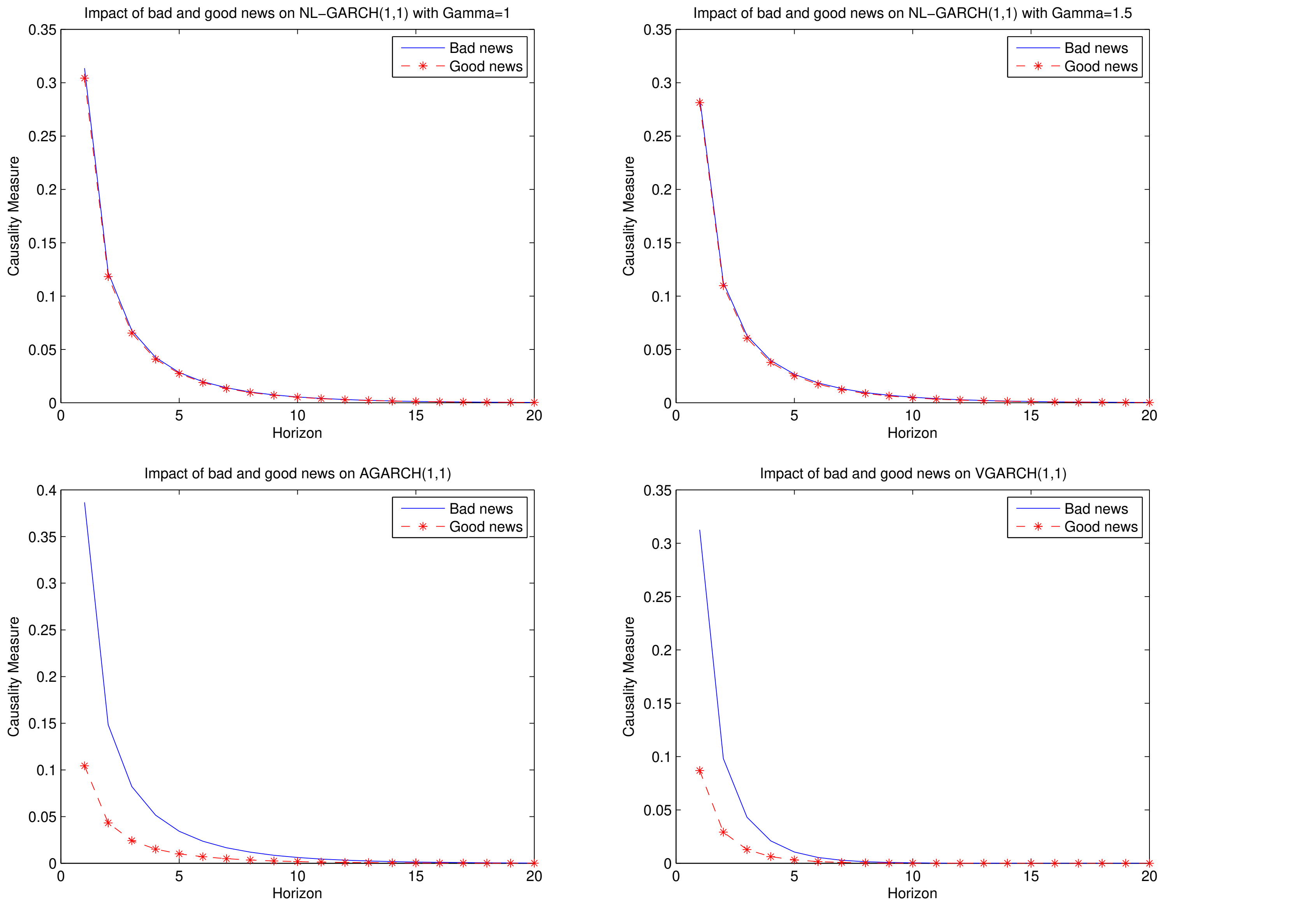


Figure 8 (continued). Causality measures of the impact of bad and good news on symmetric and asymmetric GARCH volatility models.

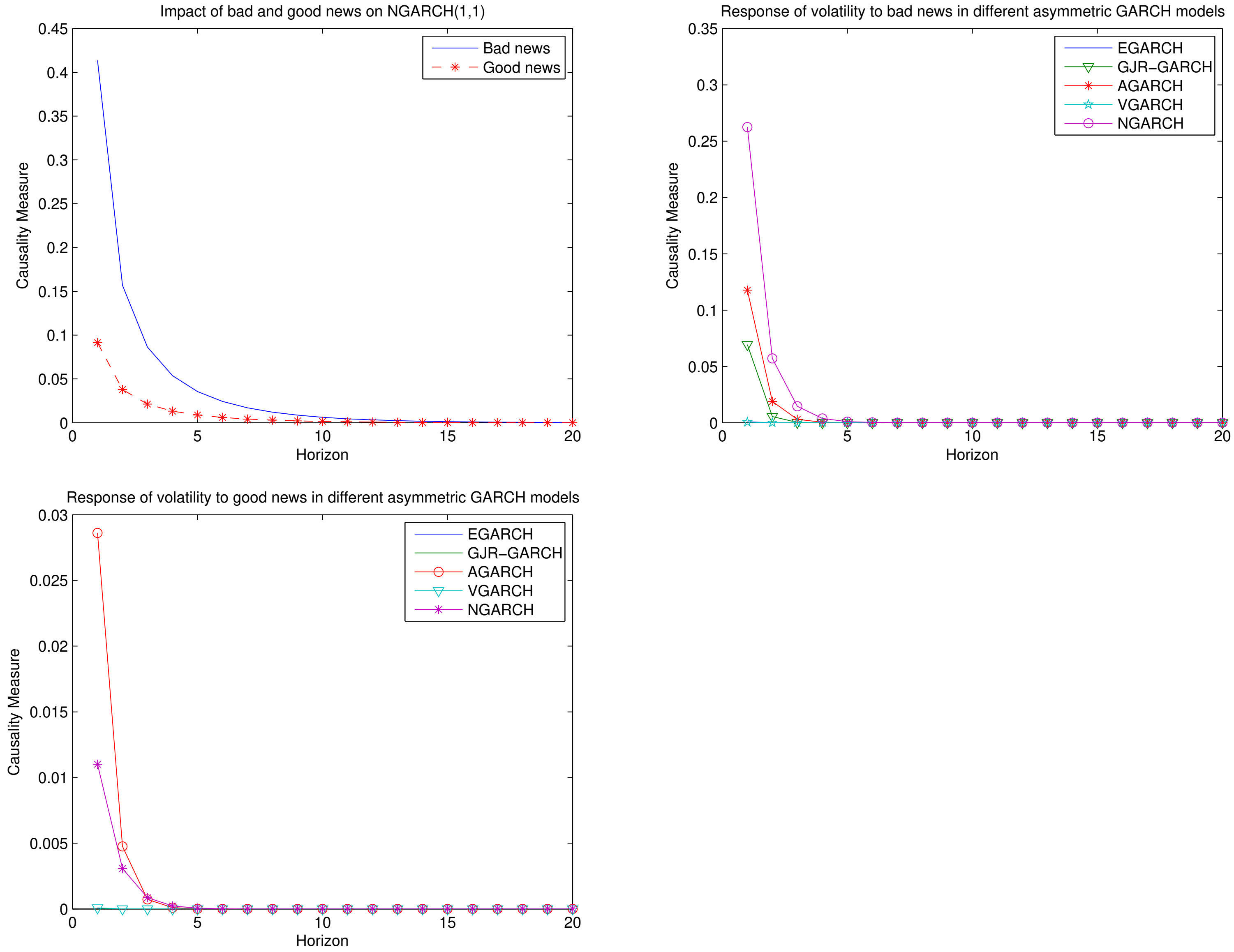


Figure 9. Causality measures of the impact of bad and good news on volatility, based on realized volatility $[\ln(RV)]$ and bipower variation $[\ln(BV)]$. January 1988 to December 2005.

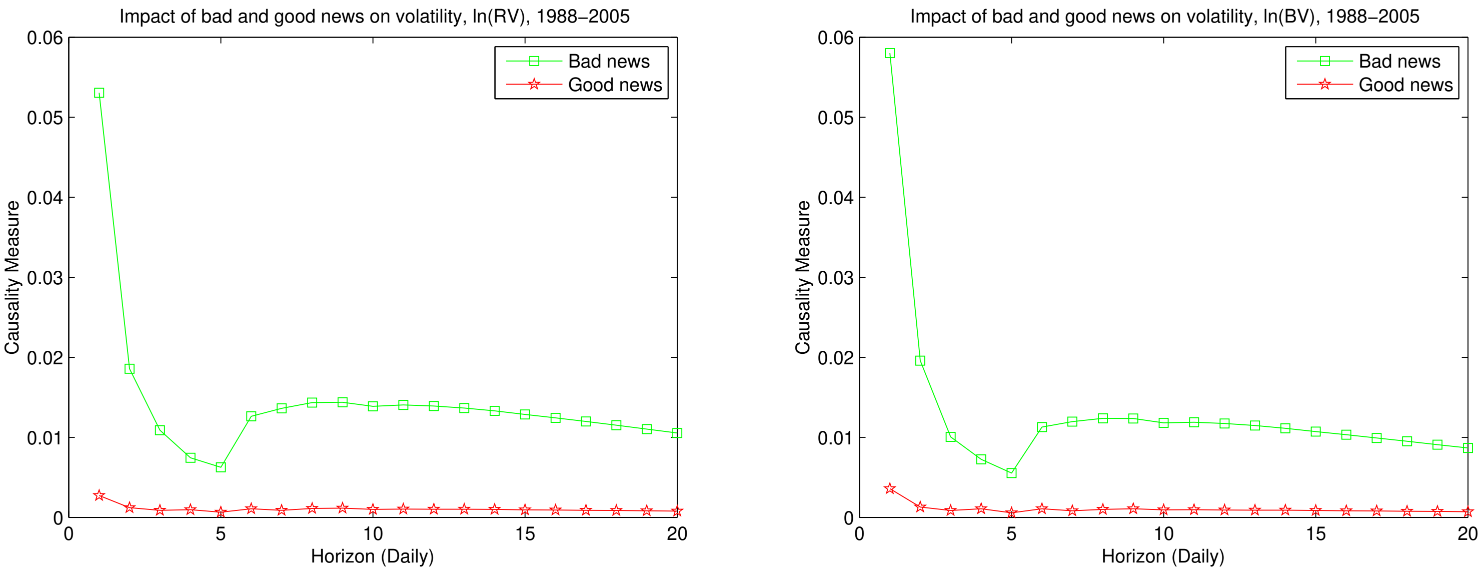
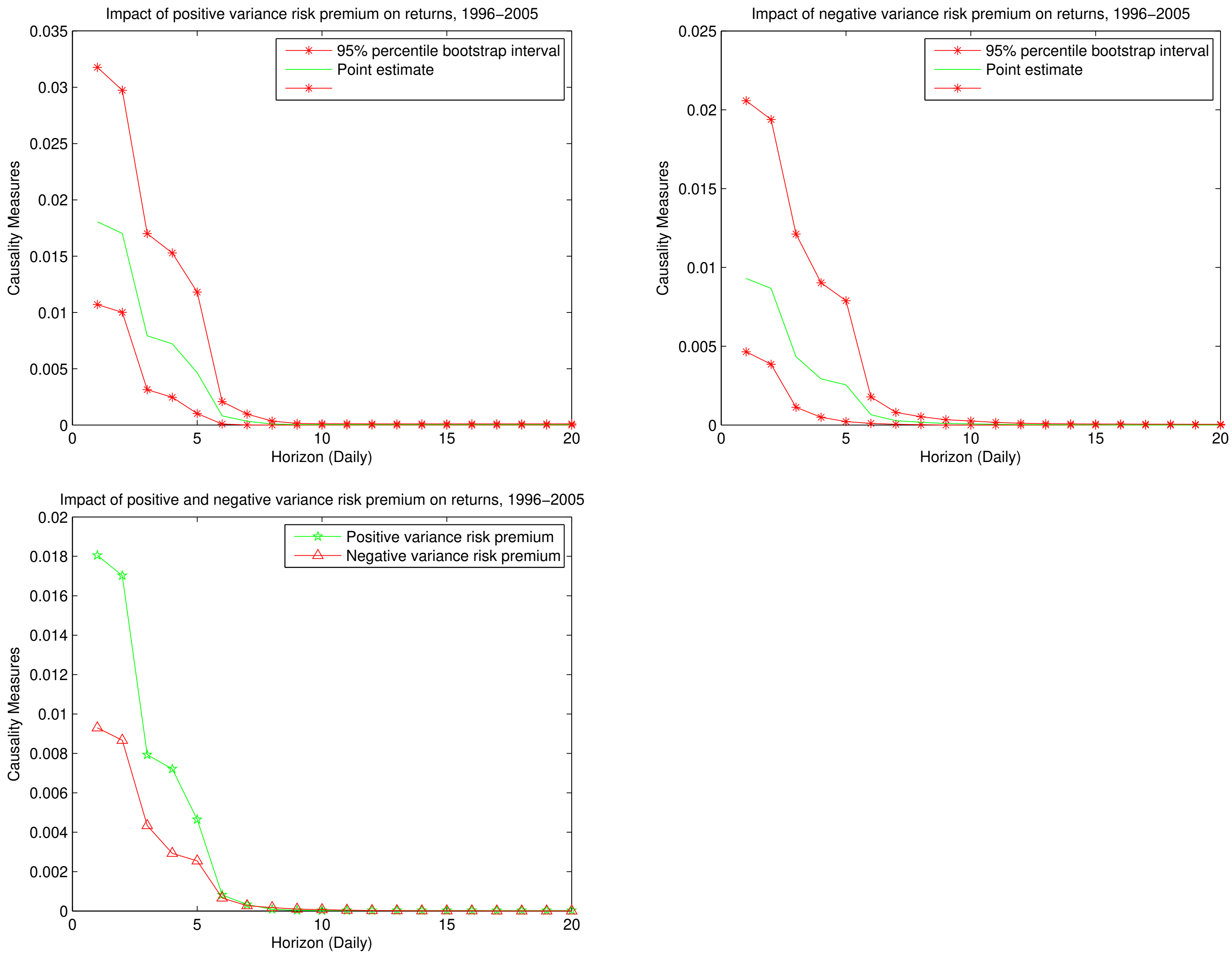


Figure 10. Causality measures of the impact of positive and negative variance risk premium on returns. January 1996 to December 2005.



C. Detailed empirical results: Point estimates and confidence intervals

Figure 11. Leverage effects in hourly and daily data, using bivariate models for $(r, \ln(RV))$ and $(r, \ln(BV))$. January 1988 to December 2005.

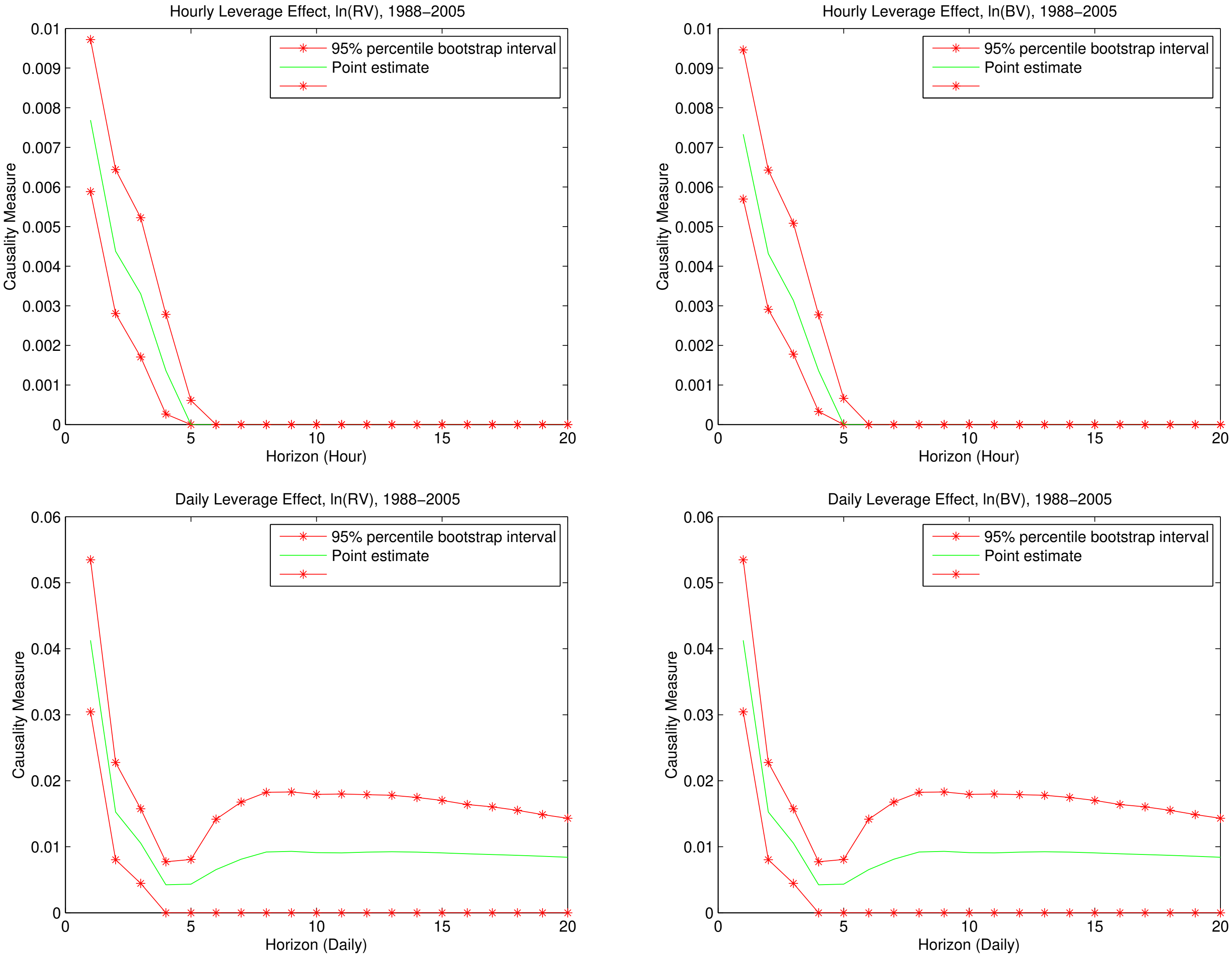


Figure 12. Instantaneous causality and dependence between daily returns and volatility using bivariate models for $(r, \ln(RV))$ and $(r, \ln(BV))$. January 1988 to December 2005.

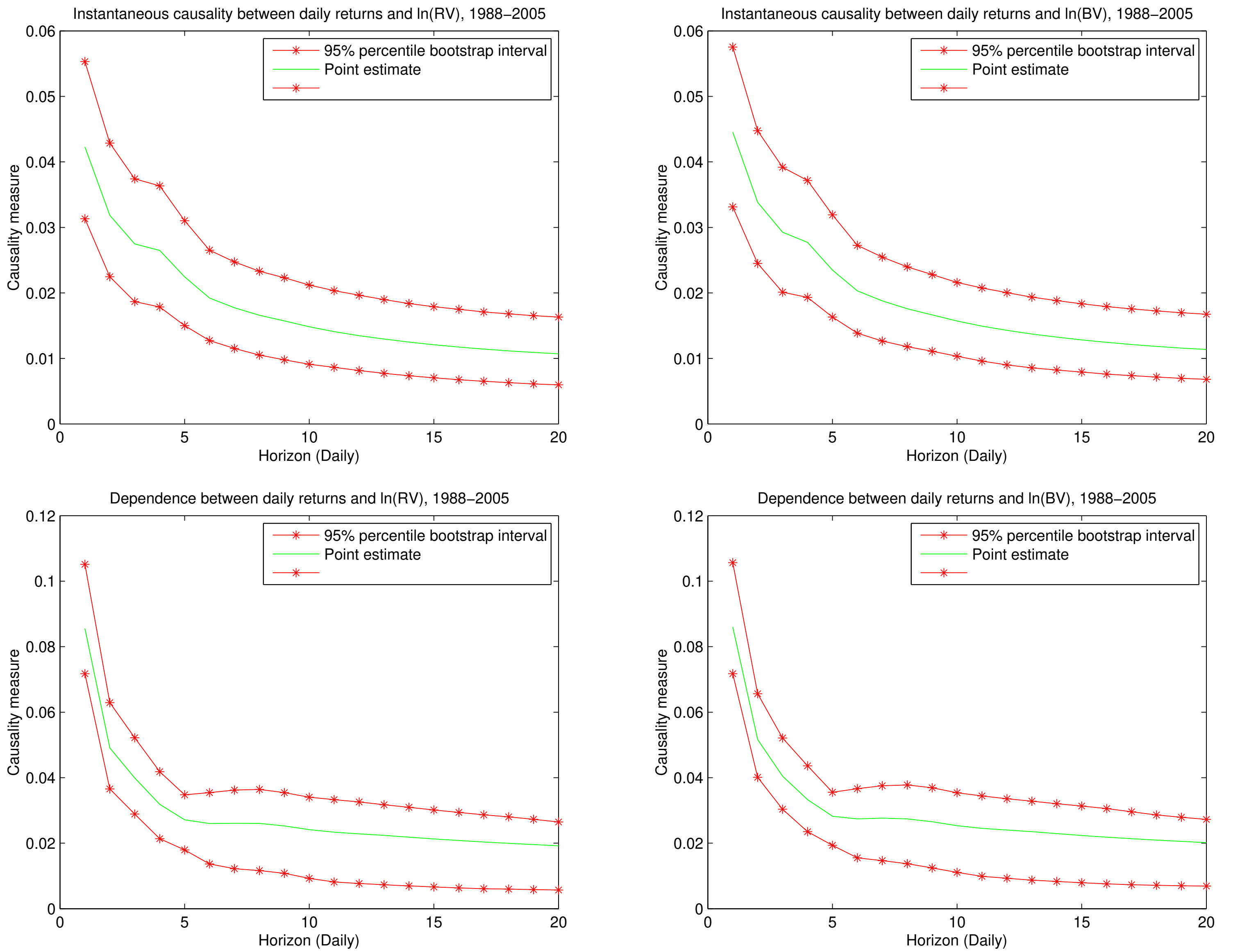


Figure 13. Causality measures between implied volatility (IV) [or variance risk premium $IV - RV$] and realized volatility (RV), using trivariate VAR models for (r, RV, IV) and $(r, RV, IV - RV)$. January 1996 to December 2005.

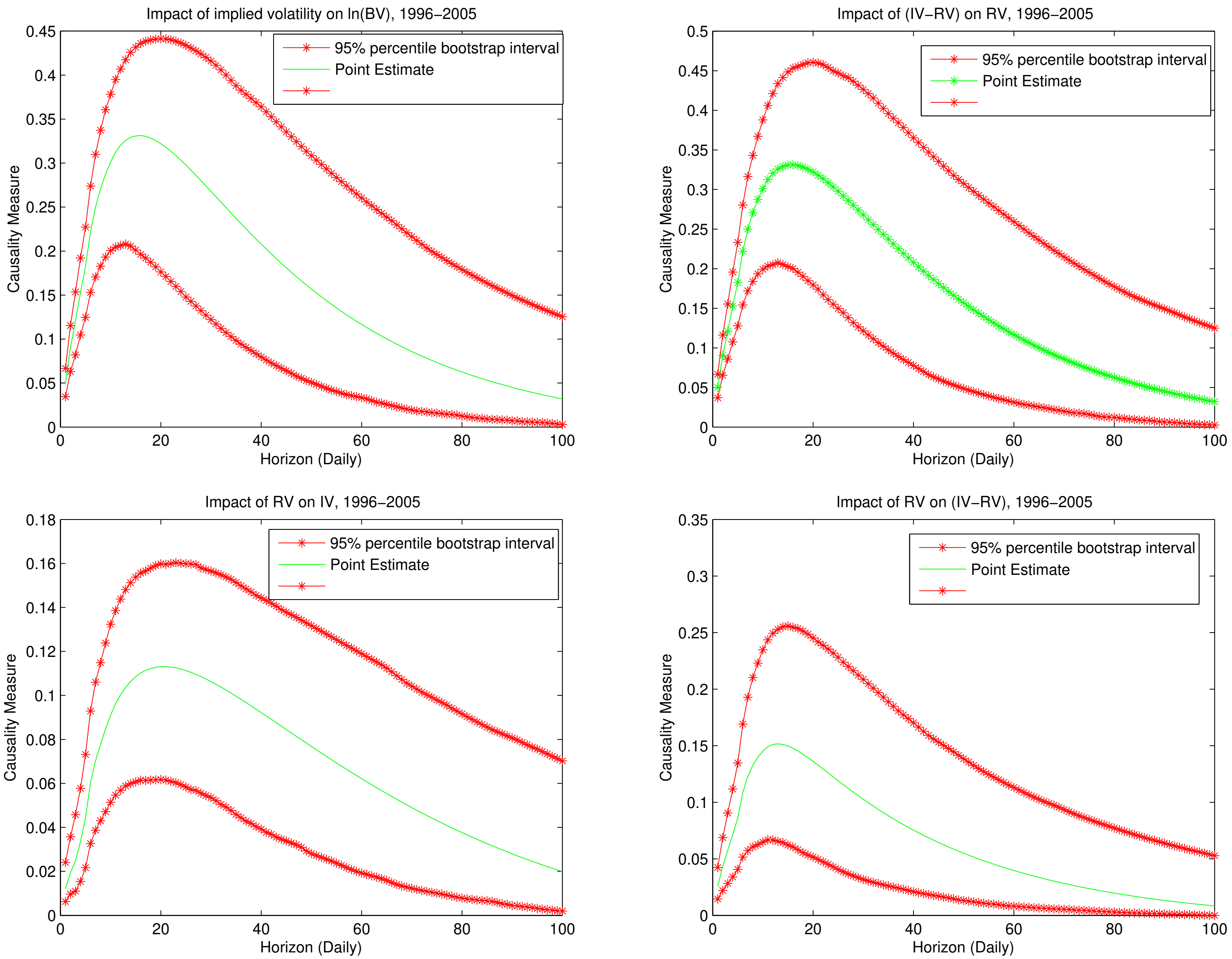


Figure 14. Volatility feedback effects, with implied volatility as auxiliary variable [trivariate model $(r,\ RV,\ IV)$] and without implied volatility [bivariate model $(r,\ RV)$]. January 1996 to December 2005.

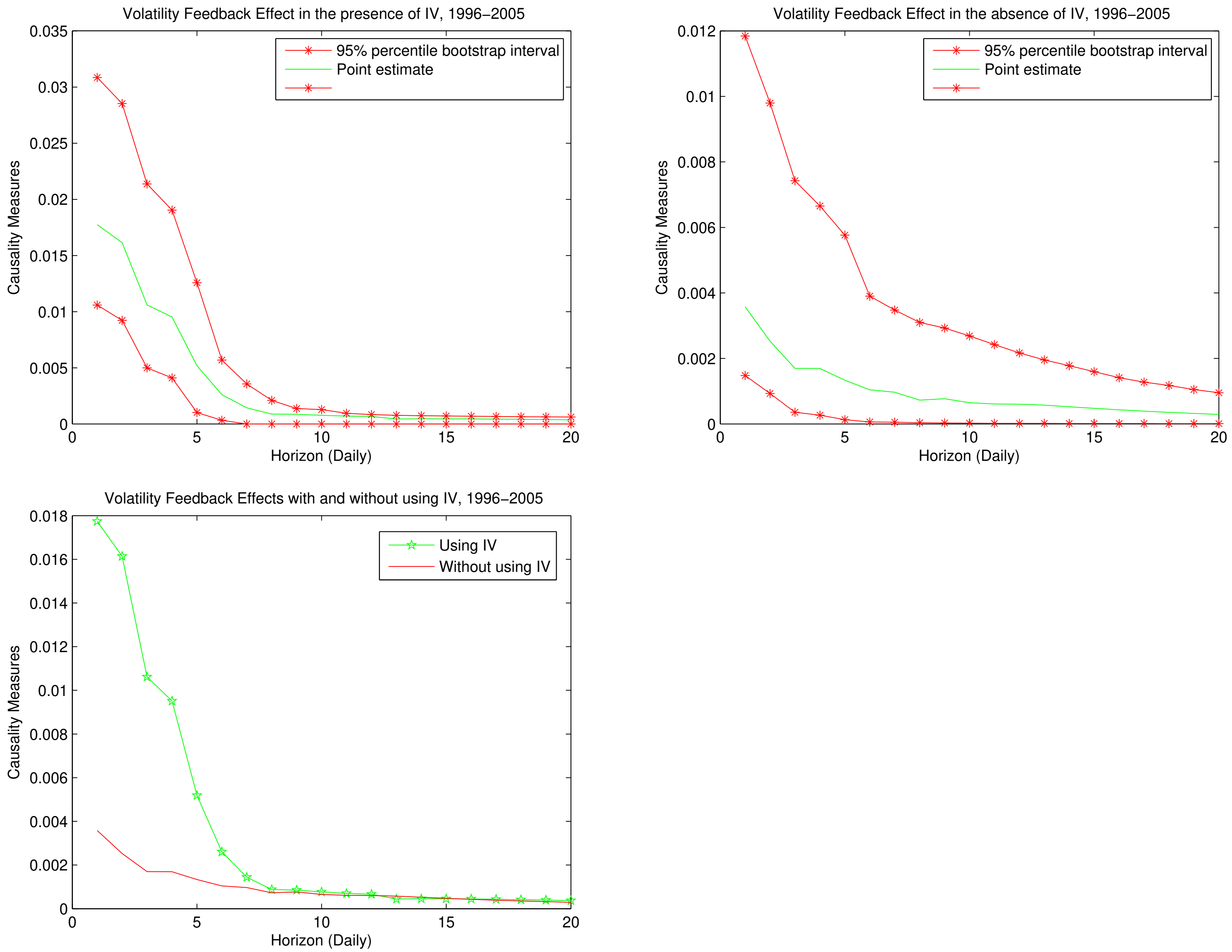


Figure 15. Other volatility feedback effects using variance risk premium $(IV - RV)$ and impact of $(RV, IV - RV)$ on returns. January 1996 to December 2005.

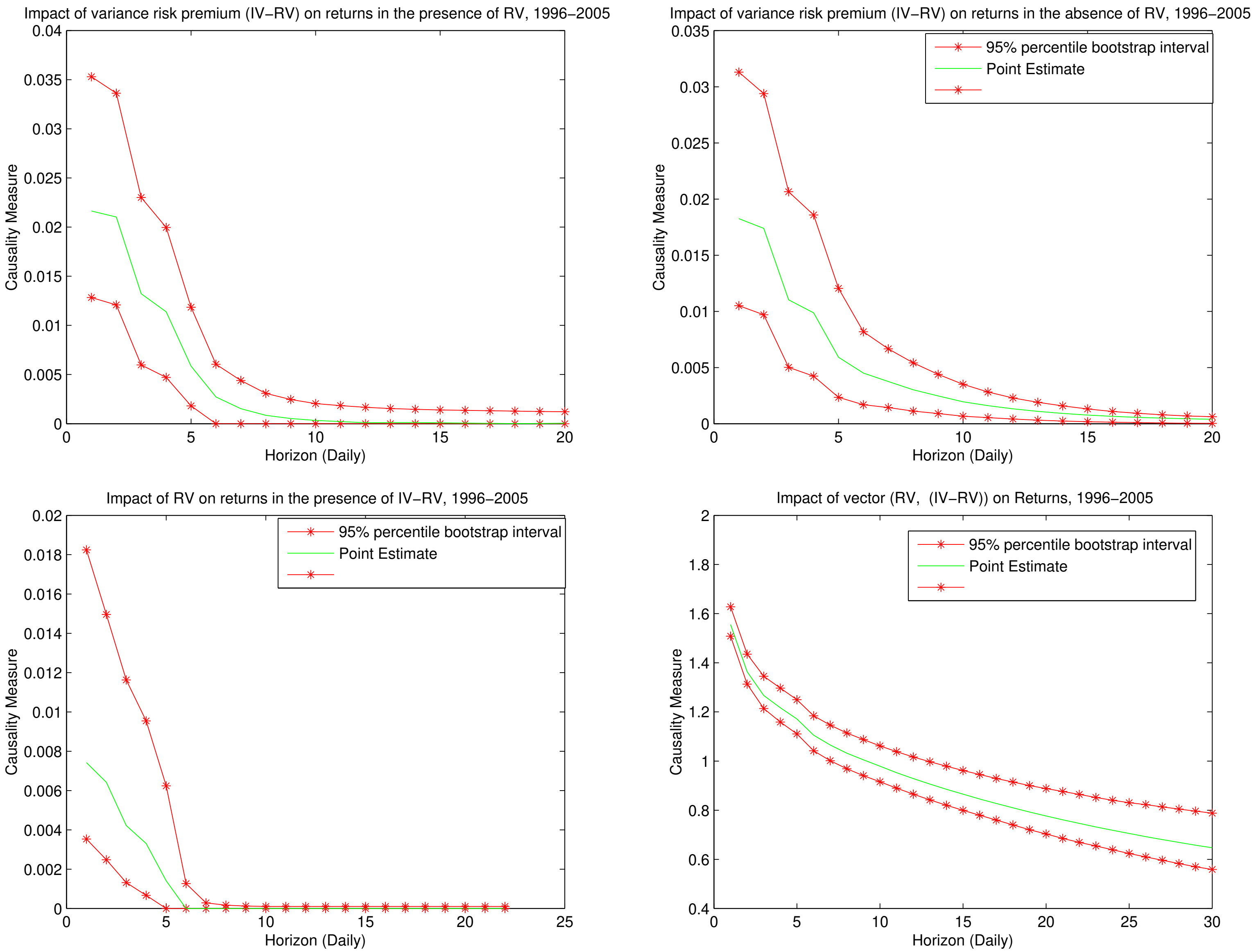


Figure 16. Leverage effects, with implied volatility as auxiliary variable [trivariate model $(r,\ RV,\ IV)$ or $(r,\ RV,\ IV-RV)$] and without implied volatility [bivariate model $(r,\ RV)$]. January 1996 to December 2005.

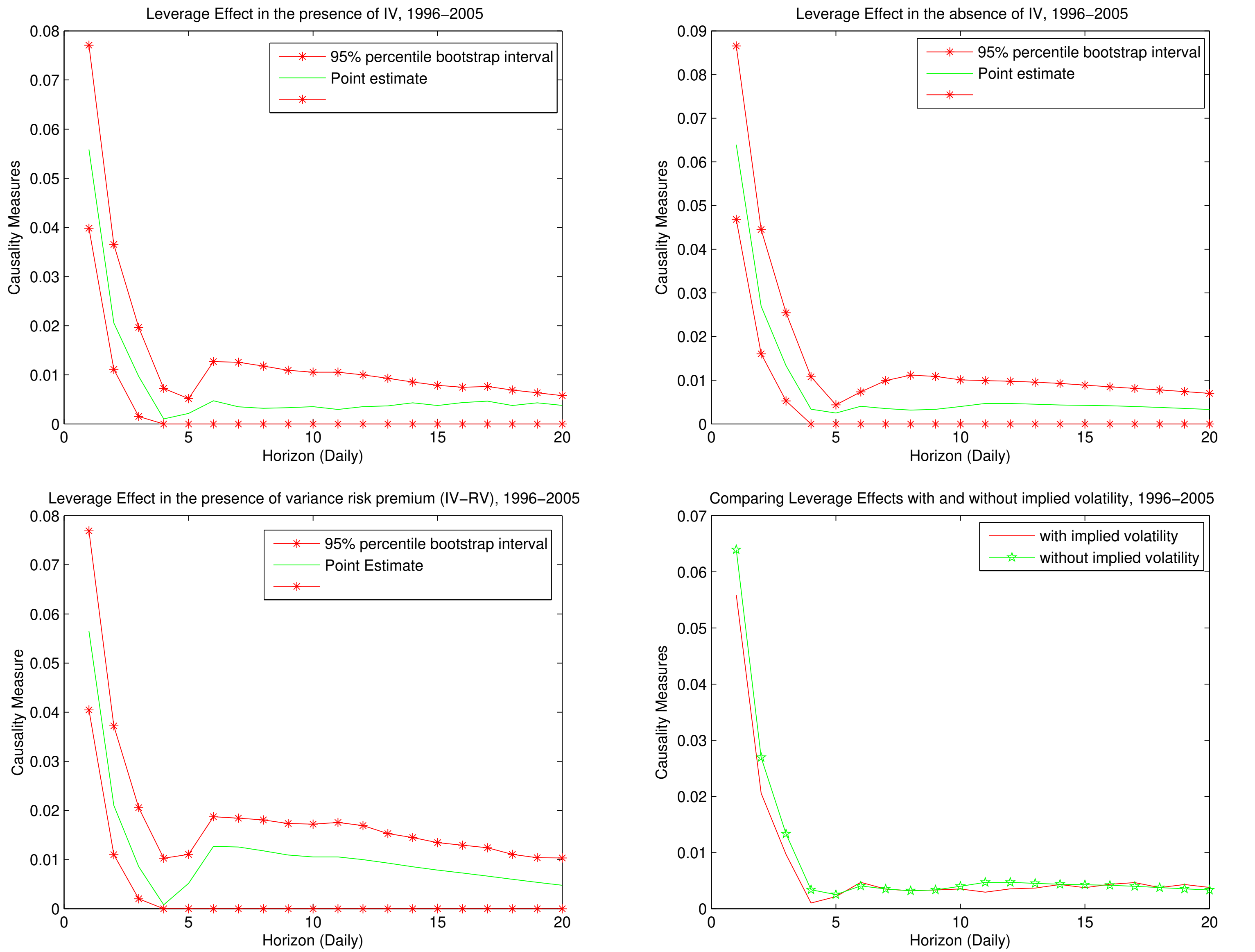


Figure 17. Causality measures of the impact of bad news on volatility (CNIF), using 5 estimators of the conditional mean ($m = 15$, 30, 90, 120, 240), realized volatility $[\ln(RV)]$ and bipower variation $[\ln(BV)]$. January 1988 to December 2005.

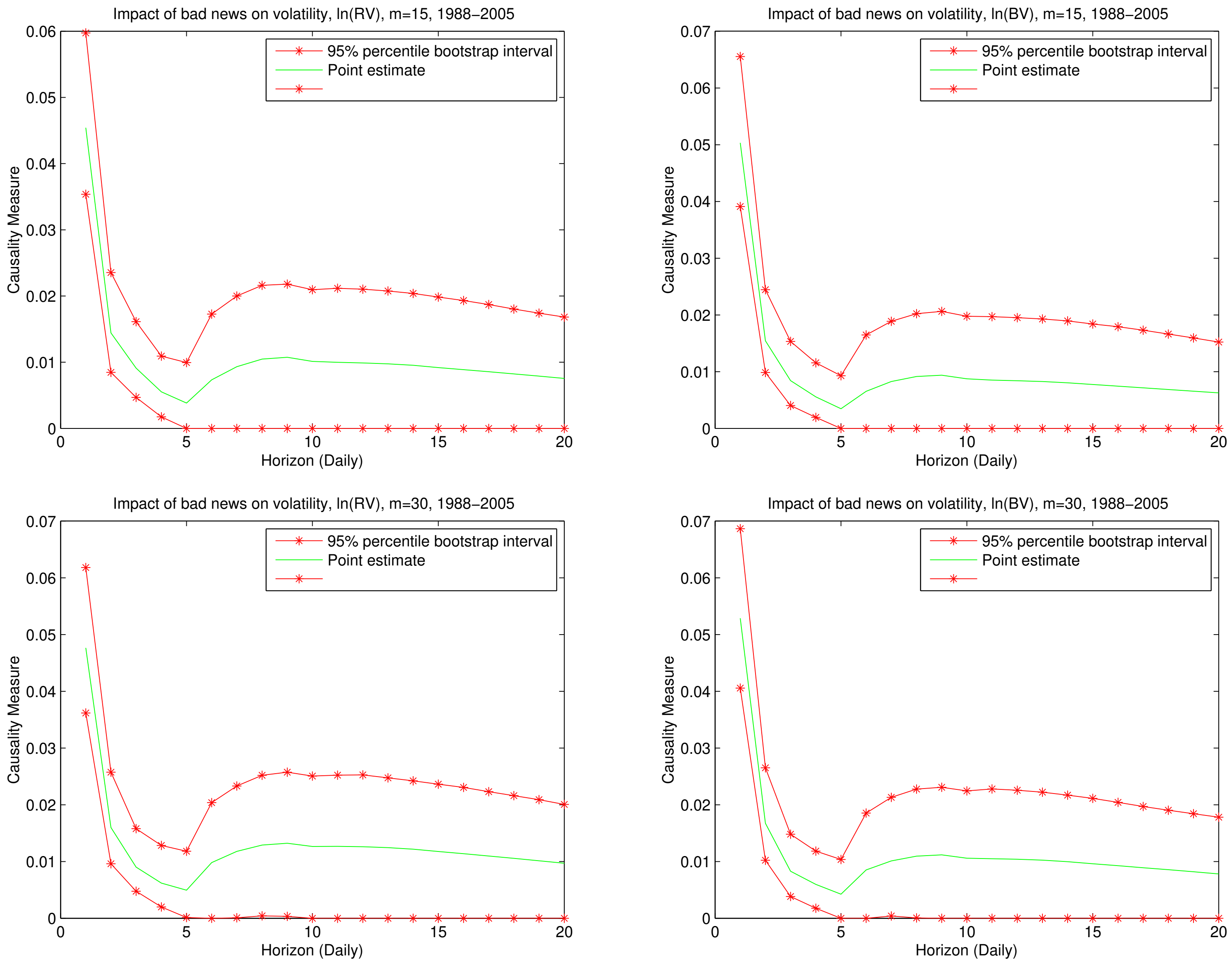


Figure 17 (continued). Causality measures of the impact of bad news on volatility (CNIF), using 5 estimators of the conditional mean ($m = 15, 30, 90, 120, 240$), realized volatility $[\ln(RV)]$ and bipower variation $[\ln(BV)]$. January 1988 to December 2005.

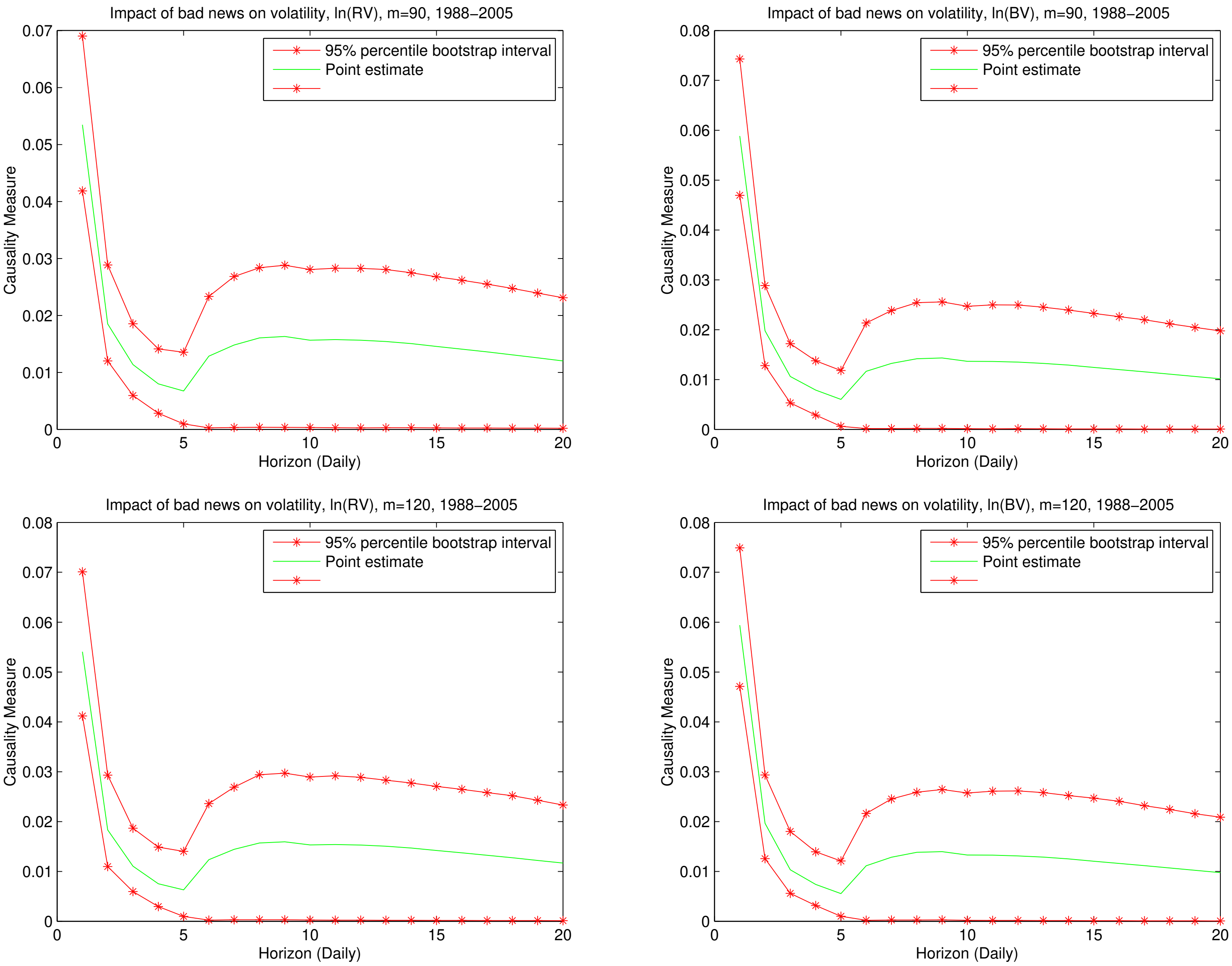


Figure 17 (continued). Causality measures of the impact of bad news on volatility (CNIF), using 5 estimators of the conditional mean $(m = 15, 30, 90, 120, 240)$, realized volatility $[\ln(RV)]$ and bipower variation $[\ln(BV)]$. January 1988 to December 2005.