# Identification and causality in macroeconomics and finance [*]

Jean-Marie Dufour [†]

June 2008
This version: June 8, 2008, 6:23am

[*] This work was supported by the Canada Research Chair Program (Chair in Econometrics, Université de Montréal), the Canadian Network of Centres of Excellence [program on *Mathematics of Information Technology and Complex Systems* (MITACS)], the Canada Council for the Arts (Killam Fellowship), the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada, and the Fonds FCAR (Government of Québec).

[†] Canada Research Chair Holder (Econometrics). Centre de recherche et développement en économique (C.R.D.E.), Centre interuniversitaire de recherche en analyse des organisations (CIRANO), and Département de sciences économiques, Université de Montréal. Mailing address: Département de sciences économiques, Université de Montréal, C.P. 6128 succursale Centre-ville, Montréal, Québec, Canada H3C 3J7. TEL: 1 514 343 2400; FAX: 1 514 343 5831; e-mail: jean.marie.dufour@umontreal.ca. Web page: http://www.fas.umontreal.ca/SCECO/Dufour .

**Contents**

# 1. Introduction

Research on issues related to identification (structural modelling) and causality.

## 1. Identification
## 1.1 Theoretical issues

Doko Tchatoka, F. and Dufour, J.-M. (2008), 'Instrument endogeneity and identification-robust tests: Some analytical results', *Journal of Statistical Planning and Inference* **138**, 2649–2661.

Dufour, J.-M. (1997), 'Some impossibility theorems in econometrics, with applications to structural and dynamic models', *Econometrica* **65**, 1365–1389.

Dufour, J.-M. (2003), 'Identification, weak instruments and statistical inference in econometrics', *Canadian Journal of Economics* **36**(4), 767–808.

Dufour, J.-M. and Jasiak, J. (2001), 'Finite sample limited information inference methods for structural equations and models with generated regressors', *International Economic Review* **42**, 815–843.

Dufour, J.-M. and Taamouti, M. (2003*a*), On methods for selecting instruments, Technical report, C.R.D.E., Université de Montréal.

Dufour, J.-M. and Taamouti, M. (2003*b*), Point-optimal instruments and generalized Anderson-Rubin procedures for nonlinear models, Technical report, C.R.D.E., Université de Montréal.

Dufour, J.-M. and Taamouti, M. (2005), 'Projection-based statistical inference in linear structural models with possibly weak instruments', *Econometrica* **73**(4), 1351–1365.

Dufour, J.-M. and Taamouti, M. (2007), 'Further results on projection-based inference in IV regressions with weak, collinear or missing instruments', *Journal of Econometrics* **139**(1), 133–153.

## 1.2 Applications

### 1.2.1 New Keynesian Phillips curves

Dufour, J.-M., Khalaf, L. and Kichian, M. (2006*a*), 'Inflation dynamics and the New Keynesian Phillips curve: An identification robust econometric analysis', *Journal of Economic Dynamics and Control* **30**(9-10), 1707–1727.

Dufour, J.-M., Khalaf, L. and Kichian, M. (2006*b*), Structural multi-equation macroeconomic models: A system-based estimation and evaluation approach, Technical report, McGill University (Department of Economics), Montréal, and Bank of Canada, Ottawa, Canada, Montréal, Canada.

Dufour, J.-M., Khalaf, L. and Kichian, M. (2007), Which NKPC? Identification-robust estimates of price stickiness and real wage rigidity, Technical report, McGill University (Department of Economics), Montréal, and Bank of Canada, Ottawa, Canada.

Dufour, J.-M., Khalaf, L. and Kichian, M. (2008), How much do real wage rigidities matter for Canadian inflation?, Technical report, McGill University (Department of Economics), Montréal, and Bank of Canada, Ottawa, Canada, Montréal, Canada.

### 1.2.2 Capital asset pricing models

Beaulieu, M.-C., Dufour, J.-M. and Khalaf, L. (2006), Testing Black's CAPM with possibly non-gaussian errors: An exact identification-robust simulation-based approach, Technical report, CIREQ, Université de Montréal and Université Laval.

Beaulieu, M.-C., Dufour, J.-M. and Khalaf, L. (2007), 'Finite-sample multivariate tests of asset pricing models with coskewness', *Computational Statistics and Data Analysis* **forthcoming**.

### 1.2.3 Growth

Dufour, J.-M. and Taamouti, M. (2007), ‘Further results on projection-based inference in IV regressions with weak, collinear or missing instruments’, *Journal of Econometrics* **139**(1), 133–153.

# 2. Causality

## 2.1 Causality in VARMA models


Boudjellaba, H., Dufour, J.-M. and Roy, R. (1992), 'Testing causality between two vectors in multivariate ARMA models', *Journal of the American Statistical Association* **87**(420), 1082–1090.

Boudjellaba, H., Dufour, J.-M. and Roy, R. (1994), 'Simplified conditions for non-causality between two vectors in multivariate ARMA models', *Journal of Econometrics* **63**, 271–287.

Dufour, J.-M. and Jouini, T. (2006), 'Finite-sample simulation-based tests in VAR models with applications to Granger causality testing', *Journal of Econometrics* **135**(1-2), 229–254.

Dufour, J.-M., Nsiri, S. and Tessier, D. (1994), Parsimonious autoregressive conditions for non-causality in multivariate ARMA models, *in* 'Proceedings of the Business and Economic Statistics Section of the American Statistical Association', Washington (D.C.), pp. 129–134.

Dufour, J.-M. and Pelletier, D. (2005), Practical methods for modelling weak VARMA processes: Identification, estimation and specification with a macroeconomic application, Technical report, Département de sciences économiques and CIREQ, Université de Montréal, Montréal, Canada.

Dufour, J.-M. and Tessier, D. (1997*a*), 'La causalité entre la monnaie et le revenu: une analyse de causalité basée sur un modèle ARMA-échelon', *L'Actualité économique* **73**, 351–366.

Dufour, J.-M. and Tessier, D. (1997*b*), La causalité entre la monnaie et le revenu: une analyse de causalité basée sur un modèle ARMA-échelon, *in* C. Gouriéroux and C. Montmarquette, eds, 'Économétrie appliquée', Economica, Paris.

## 2.2 Relationship between causality and impulse responses

Dufour, J.-M. and Renault, E. (1998), 'Short-run and long-run causality in time series: Theory', *Econometrica* **66**, 1099–1125.

Dufour, J.-M. and Tessier, D. (1993), 'On the relationship between impulse response analysis, innovation accounting and Granger causality', *Economics Letters* **42**, 327–333.

## 2.3 Short-run and long-run causality

Dufiour, J.-M. and Tessier, D. (2006), Short-run and long-run causality between monetary policy variables and stock prices, Technical Report 2006-39, Bank of Canada, Ottawa, Canada.

Dufour, J.-M., Garcia, R. and Taamouti, A. (2008), Measuring causality between volatility and returns with high-frequency data, Technical report, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal, Montréal, Canada.

Dufour, J.-M., Pelletier, D. and Renault, É. (2006), 'Short run and long run causality in time series: Inference', *Journal of Econometrics* **132**(2), 337–362.

Dufour, J.-M. and Renault, E. (1998), 'Short-run and long-run causality in time series: Theory', *Econometrica* **66**, 1099–1125.

Dufour, J.-M. and Taamouti, A. (2008), Short and long run causality measures: Theory and inference, Technical report, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal, Montréal, Canada.

## 2.3 Applications

### 2.3.1 Small macroeconomic models

### 2.3.2 Finance

## 2. Basic points and pitfalls

1. **Indirect relations** – Statistical methods for studying simultaneous equations and causality both aim at
   analyzing and avoiding being fooled by indirect relationships:

   (a) simultaneous indirect relations (standard simultaneous equations);

   (b) intertemporal indirect relations (Granger-Wiener causality).

2. **Relative nature of simultaneity and causality** – Statements on the presence of "simultaneity biases" and "causality" are always **relative**:

   (a) a set of conditioning variables (instruments);

   (b) an information set (Granger-Wiener causality).

3. **Consequences** of the above point include the following.

   (a) It is generally meaningless to claim that estimators of a linear structural equation (“IV regression”) without specifying a set of instruments which are declared to be instruments by convention (on a priori grounds);
   changing the set of instruments typically involve changing the object of interest,

   (b) Granger-Wiener causality theoretically depends on:

      i. the model used;
      ii. the aggregation of variables;
      iii. time aggregation (observation frequency).

   Changing any of these features can lead to changes in causality structures. This should be viewed as normal.

   (c) Filtering can easily distort dynamic relations and causal relations.

   (d) The time-honoured distinction between “correlation” and “causality” can easily be misleading.

In the end, everything can be reduced to predictive relationships and/or correlations.

4. **Impulse responses and causality** – Impulse responses constitute partial – possibly misleading – representations of causality properties at various horizons.

5. **Statistical and economic significance** – In statistical analysis it is important to look at both statistical significance and "economic" significance. The importance of causal links should be assessed not just tested.

6. **Statistical inference when identification may fail** – In models involving identification difficulties, many standard statistical procedures – such a standard errors and asymptotic approximations – can be highly misleading. The problem, however, can be corrected.

7. **Statistical inference in large models –** Large-sample approximations tend to be very unreliable in systems which involve many variables and parameters (VAR models, VARMA models0.
Simulation-based statistical procedures (Monte

Carlo tests, bootstrapping) tend be very helpful in such contexts.

As far as possible, inferences based on asymptotic distributions should be controlled or replaced by simulation-based procedures.

## 3. Simultaneity and indirect relations

Let

$$y_t = Y_t'\beta + u_t\,,\ t,\ \ldots\ , T. \tag{3.1}$$

If $\beta$ is defined as the regression coefficient of $y_t$ on $Y_t$, (the best linear prediction of $y_t$ based on $Y_t$), i.e.

$$P_L[y_t \,|\, Y_t] = Y_t'\beta\,, \tag{3.2}$$

then

$$C[Y_t,\, u_t] = 0 \tag{3.3}$$

and $\beta$ can be estimated by least squares.
If we suppose instead that

$$y_t \ = \ P_L[y_t \,|\, z_t] + v_{1t} = z_t'\pi_2 + v_{1t}\,, \tag{3.4}$$

$$Y_t \ = \ P_L[Y_t \,|\, z_t] + v_{2t} \ = z_t'\mathit{\Pi}_2 + v_{2t}\,, \tag{3.5}$$

we have, by construction,

$$\mathsf{C}[z_t,\, v_{1t}] = 0,\ \mathsf{C}[z_t,\, v_{2t}] = 0\,. \tag{3.6}$$

Then

$$\begin{aligned} y_t \ &= \ Y_t'\beta + u_t \\ &= \ (z_t'\mathit{\Pi}_2 + v_{2t})'\beta + u_t \\ &= \ z_t'\mathit{\Pi}_2\beta + (v_{2t}'\beta + u_t) \end{aligned}$$

$$= z_t' \pi_2 + v_{1t} \tag{3.7}$$

hence

$$\pi_2 = \Pi_2 \beta \,, \tag{3.8}$$

$$v_{1t} = v_{2t}' \beta + u_t \,, \tag{3.9}$$

$$u_t = v_{1t} - v_{2t}' \beta \,, \tag{3.10}$$

$$\mathsf{C}[z_t,\, u_{1t}] = 0 \,. \tag{3.11}$$

If $Y_t$ and $v_{2t}$ are univariate,

$$\begin{aligned} E[u_t Y_t] &= E[u_t v_{2t}] \\ &= V(v_{1t}) - \mathsf{C}[v_{1t},\, v_{2t}]\beta \neq 0 \,. \end{aligned} \tag{3.12}$$

Due to the introduction of the equations (3.4) - (3.5), the structural equation (3.1) cannot be consistently estimated by least squares.
This situation depends crucially on the decision to condition on the $z_t$ (instruments).
By changing the vector $z_t$, the interpretation and the value of $\beta$ will change (convention).

Under the above assumptions, we can also consider the regression:

$$u_t = v_{2t}'a + e_t\,,\ \mathsf{C}[e_t,\ v_{2t}] = 0, \tag{3.13}$$

hence

$$\begin{aligned} y_t &= Y_t'\beta + u_t \\ &= Y_t'\beta + v_{2t}'a + e_t \end{aligned} \tag{3.14}$$

where, by construction,

$$\mathsf{C}[e_t,\ v_{2t}] = 0\,,\ \mathsf{C}[e_t, Y_t] = 0\,. \tag{3.15}$$

Equation (3.14) is a regression equation.

The problem here is that $v_{2t}$ is not observable. If

$$v_{2t} \text{ is replaced by}$$
$$\hat{v}_{2t} = Y_t - z_t'\hat{\Pi}_2$$

where $\hat{\Pi}_2$ is the least squares estimator of $\Pi_2$ based on regressing $Y_t$ on $z_t$, the least squares estimator of $\beta$ from the approximate equation

$$y_t = Y_t'\beta + \hat{v}_{2t}'a + e_t^*\,,\ t,\ \ldots\ ,\ T\ , \tag{3.16}$$

is the 2SLS estimator of $\beta$, while $a$ provides information on the endogeneity of $Y_t$ in equation (3.1).

The $F$ test for

$$H_0 : a = 0 \tag{3.17}$$

is a variant of the Durbin-Wu-Hausman test of exogeneity.

Equation (3.16) is also equivalent to

$$y_t = \hat{Y}_t' \beta + \hat{v}_{2t}' a_* + e_t^* \,,\ t, \ \ldots \ , T \ , \tag{3.18}$$

where $\hat{Y}_t = z_t' \hat{\Pi}_2$ and $a_* = a + \beta$.

The 2SLS estimator works provided

$$\Pi_2 \text{ has full column rank.} \tag{3.19}$$

This is called the **rank condition for identification**, because $\beta$ must be determined by solving the equation

$$\pi_2 = \Pi_2 \beta \,, \tag{3.20}$$

If identification does not hold, equation (3.18) involves an

$$\textbf{asymptotic collinearity}. \tag{3.21}$$

This brings us to the topic of identification and weak identification.

# 4. Nonidentification and weak identification

In the linear structural model discussed above, the structural parameter $\beta$ is determined by solving the equation

$$\pi_2 = \mathit{\Pi}_2 \beta \tag{4.22}$$

for $\beta$. The latter has a unique solution in terms of the regression coefficients $\pi_2$ and $\mathit{\Pi}_2$ if and only if

$$\mathit{\Pi}_2 \text{ has full column rank.} \tag{4.23}$$

In practice, even if the identification condition holds, it appears to be often the case that structural parameters like $\beta$ are "close" not to be identifiable. This can be made more precise by saying that

- $\det(\mathit{\Pi}_2' \mathit{\Pi}_2)$ is "close to zero",
  or (equivalently)
- $\mathit{\Pi}_2' \mathit{\Pi}_2$ has one or several eigenvalues "close to zero".

Several authors in the pas have noted that usual asymptotic approximations are not valid or lead to very inaccurate results when parameters of interest are close to regions where these parameters are not anymore identifiable:

Sargan (1983, Econometrica)
Phillips (1984, International Economic Review)
Phillips (1985, International Economic Review)
Gleser and Hwang (1987, Annals of Statistics)
Koschat (1987, Annals of Statistics)
Phillips (1989, Econometric Theory)
Hillier (1990, Econometrica)
Nelson and Startz (1990a, Journal of Business)
Nelson and Startz (1990b, Econometrica)
Buse (1992, Econometrica)
Maddala and Jeong (1992, Econometrica)
Choi and Phillips (1992, Journal of Econometrics)
Bound, Jaeger, and Baker (1993, NBER Discussion Paper)
Dufour and Jasiak (1993, CRDE)
Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association)

McManus, Nankervis, and Savin (1994, Journal of Econometrics)

Hall, Rudebusch, and Wilcox (1996, International Economic Review)

Dufour (1997, Econometrica)

Shea (1997, Review of Economics and Statistics)

Staiger and Stock (1997, Econometrica)

Wang and Zivot (1998, Econometrica)

Zivot, Startz, and Nelson (1998, International Economic Review)

Startz, Nelson, and Zivot (1999, International Economic Review)

Perron (1999)

Stock and Wright (2000, Econometrica)

Dufour and Jasiak (2001, International Economic Review)

Dufour and Taamouti (2001)

Kleibergen (2001, 2002)

Moreira (2001, 2002)

Stock and Yogo (2002)

Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)

Dufour (2003, Canadian Journal of Economics)

Dufour and Taamouti (2005, Econometrica)

Dufour and Taamouti (2006, Journal of Econometrics, forth.)

Surveys:

- Stock, Wright, and Yogo (2002, Journal of Business and Economic Statistics)
- Dufour (2003, Canadian Journal of Economics)

Weak instruments have been notorious to cause serious statistical difficulties, form the viewpoints of:

1. estimation;
2. confidence interval construction;
3. testing.

## Difficulties

1. Theoretical results show that the distributions of various estimators depend in a complicated way upon unknown nuisance parameters. So they are difficult to interpret.
2. When identification conditions do not hold, standard asymptotic theory for estimators and test statistics typically collapses.
3. With weak instruments,
   (a) 2SLS becomes heavily biased (in the same direction as OLS),
   (b) distribution of 2SLS is quite far the normal distribution (e.g., bimodal).

4. Standard Wald-type procedures based on asymptotic standard errors become fundamentally unreliable or very unreliable in finite samples [Dufour (1997, Econometrica)].

5. Problems were strikingly illustrated by the reconsideration by Bound, Jaeger, and Baker (1995, Journal of the American Statistical Association) of a study on returns to education by Angrist and Krueger (1991, QJE):

   - 329000 observations;
   - replacing the instruments used by Angrist and Krueger (1991, QJE) with randomly generated instruments (totally irrelevant) produced very similar point estimates and standard errors;
   - indicates that the instruments originally used were weak.

Crucial to use finite-sample approaches to produce reliable inference.

Finite-sample approaches to inference on models involving weak identification

- Dufour (1997, Econometrica)
- Dufour and Jasiak (2001, International Economic Review)
- Dufour and Taamouti (2005, Econometrica)
- Beaulieu, Dufour, and Khalaf (2005)
- Dufour and Valéry (2005)
- Dufour and Taamouti (2006, Journal of Econometrics, forth.)
- Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control)
- Dufour, Khalaf, and Kichian (2006b)
- Dufour, Khalaf, and Kichian (2006d)

## Important features

1. Procedures robust to lack of identification (or weak identification)
2. Procedures for which a finite-sample distributional theory can be supplied, at least in some reference cases
3. Limited information methods which do not require a complete formulation of the model [limited-information vs. full-information methods]
   (a) Robustness to missing instruments
   (b) Robustness to the formulation of the model for the explanatory endogenous variables

## 5. Weak identification and New Keynesian Phillips Curves

For basic NKPC, the issue of weak identification has been considered by several authors:

Ma (2002, Economics Letters)

Khalaf-Kichian (2004)

Mavroeidis (2004, Oxford Bulletin of Economics and Statistics)

Mavroeidis (2005, JMCB)

Yazgan-Yilmazkuday (2005, Studies in Nonlinear Dynamics and Econometrics)

Nason and Smith (2005)

Dufour, Khalaf, and Kichian (2006a, Journal of Economic Dynamics and Control)

Mavroeidis (2006)

1. Dufour, J.-M., L. Khalaf, and M. Kichian (2006a): "Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis," *Journal of Economic Dynamics and Control*, 30 (9-10), 1707–1727.

Gali-Gertler (JME, 1999) model

$$\begin{aligned}\underbrace{\pi_t}_{\text{inflation}} &= \lambda \underbrace{s_t}_{\text{marginal costs}} + \gamma_f \boxed{E_t \pi_{t+1}} + \gamma_b \, \pi_{t-1} \\ &= \lambda s_t + \gamma_f \pi_{t+1} + \gamma_b \, \pi_{t+1} + u_{t+1}\end{aligned}$$

$$\lambda = \frac{(1-\omega)(1-\theta)(1-\beta\theta)}{\theta + \omega - \omega\theta + \omega\beta\theta}$$

$$\gamma_f = \frac{\beta\theta}{\theta + \omega - \omega\theta + \omega\beta\theta}$$ ► forward-looking

$$\gamma_b = \frac{\omega}{\theta + \omega - \omega\theta + \omega\beta\theta}$$ ► backward-looking

$$\beta \equiv \text{ subjective discount rate}$$

- Identification-robust tests and CS for model parameters $(\lambda, \gamma_f, \gamma_b)$ and $(\omega, \theta, \beta)$ based on AR-type statistics and projection techniques.
- Rational and survey expectations studied.
- Survey expectations variants rejected.
- Model acceptable for the U.S. but not for Canada.

2. Dufour, J.-M., L. Khalaf, and M. Kichian (2006b): "Structural Estimation and Evaluation of Calvo-Style Inflation Models," Discussion paper, CIREQ, Un. de Montréal, and Bank of Canada.

Calvo-type inflation model studied by Eichenbaum and Fisher (2005) model.

3. Dufour, J.-M., L. Khalaf, and M. Kichian (2006c): "Structural Multi-Equation Macroeconomic Models: A System-Based Estimation and Evaluation Approach," Discussion paper, CIREQ, Un. de Montréal, and Bank of Canada.

Lindé (JME, 2005) multi-equation NKPC.

## 6. Short-run and long-run causality

It is possible that a variable Y does not cause a variable X at horizon $1$, but causes it at horizon $h > 1$ (indirect causality transmitted by an auxiliary variable $Z$)
[Lütkepohl (1993), Dufour and Renault (1998)].

$$\begin{pmatrix} X(t+1) \\ Y(t+1) \\ Z(t+1) \end{pmatrix} = \begin{bmatrix} 0.60 & 0.00 & 0.80 \\ 0.00 & 0.40 & 0.00 \\ 0.00 & 0.60 & 0.10 \end{bmatrix} \begin{pmatrix} X(t) \\ Y(t) \\ Z(t) \end{pmatrix} + \begin{pmatrix} \varepsilon_X(t+1) \\ \varepsilon_Y(t+1) \\ \varepsilon_Z(t+1) \end{pmatrix}$$

$$\Rightarrow \; X(t+1) = 0.6X(t) + 0.8Z(t) + \varepsilon_X(t+1).$$

Since the coefficient of $Y(t)$ is $0$, we can conclude that $Y$ does not cause $X$ at horizon $1$ [Wiener (1956), Granger (1969, Econometrica)].

If we consider the above model at time $(t+2)$ :

$$\begin{aligned} X(t+2) \; = \; & 0.36\, X(t) + 0.48Y(t) \\ & + 0.56\, Z(t) + 0.6\varepsilon_X(t+1) \\ & + 0.8\varepsilon_Z(t+1) + \varepsilon_X(t+2). \end{aligned}$$

The coefficient of $Y(t)$ is equal to $0.48$, which implies that $Y$ causes $X$ at horizon $2$. Here we are in presence of an indirect effect $(0.48 = 0.80 \times 0.60)$,

$$Y \stackrel{0.6}{\rightarrow} Z \stackrel{0.8}{\rightarrow} X$$

1. Processes:

   $\{X(t) : t \in \mathbb{Z}\}$ , $\{Y(t) : t \in \mathbb{Z}\}$, $\{Z(t) : t \in \mathbb{Z}\}$

   $X(t)$ and $Y(t)$ scalar, $Z(t)$ vector.

2. Information sets:

$$
\begin{aligned}
\underline{X}_t &= \{X(s), s \leq t\}, \\
\underline{Y}_t &= \{Y(s), s \leq t\}, \\
\underline{Z}_t &= \{Z(s), s \leq t\}, \\
I_t &= \underline{X}_t \cup \underline{Y}_t \cup \underline{Z}_t.
\end{aligned}
$$

3. The variance of the forecast error of $X(t+h)$ based on the information set $A_t$, for $A_t = I_t$, $I_t - \underline{Y}_t = \underline{X}_t \cup \underline{Z}_t$ :

$$\sigma^2(X(t+h) \mid A_t).$$

   Linear prediction.

**Definition 6.1** *For $h \geq 1$, we say that $Y$ does not cause $X$ at horizon $h$ given all elements of $I_t$ except the past of $Y$, denoted*

$$Y \underset{h}{\nrightarrow} X \mid Z$$

*if*

$$\sigma^2(X(t+h) \mid I_t - \underline{Y}_t) = \sigma^2(X(t+h) \mid I_t),\ \forall t \geq 0\,.$$

To be more specific, let

$$W(t) = \sum_{j=1}^{\infty} \pi_j W(t-j) + a(t) \tag{6.1}$$

Then

$$W(t) = (X(t)', Y(t)', Z(t)')' . \tag{6.2}$$

Then the best linear forecast of $W(t+h)$ given the history $I_t$ of the process at time $t$ is

$$P[W(t+h)|I_t] = \sum_{j=1}^{\infty} \pi_j^{(h)} W(t+1-j) \tag{6.3}$$

where

$$\pi_j^{(1)} = \pi_j \ , \quad \pi_j^{(h+1)} = \pi_{j+1}^{(h)} + \pi_1^{(h)} \pi_j \ , \ h = 1, 2, \ \ldots \tag{6.4}$$

Setting

$$\pi_j^{(h)} = \begin{bmatrix} \pi_{XXj}^{(h)} & \pi_{XYj}^{(h)} & \pi_{XZj}^{(h)} \\ \pi_{YXj}^{(h)} & \pi_{YYj}^{(h)} & \pi_{YZj}^{(h)} \\ \pi_{ZXj}^{(h)} & \pi_{ZYj}^{(h)} & \pi_{ZZj}^{(h)} \end{bmatrix} \tag{6.5}$$

we have:

$$Y \underset{h}{\nrightarrow} X|I_{XZ} \Leftrightarrow \pi_{XYj}^{(h)} = 0\ ,\ \forall j \in \mathbb{N}\ . \tag{6.6}$$

Let us consider the moving average representation of the process (under stationarity):

$$W(t) = \sum_{j=0}^{\infty} \psi_j a(t-j)\,. \tag{6.7}$$

Then

$$\pi_1^{(h)} = \psi_h\ , \quad \forall h \geq 0\ . \tag{6.8}$$

Setting

$$\psi_h = \begin{bmatrix} \psi_{XXh} & \psi_{XYh} & \psi_{XZh} \\ \psi_{YXh} & \psi_{YYh} & \psi_{YZh} \\ \psi_{ZXh} & \psi_{ZYh} & \psi_{ZZh} \end{bmatrix}\ ,\ h \geq 0\ , \tag{6.9}$$

the condition

$$\psi_{XYh} = 0,\ \text{for } h \geq 0 \tag{6.10}$$

is neither necessary nor sufficient for $Y \underset{h}{\nrightarrow} X|I_{XZ}$.

## Example of discrepancy between impulse responses and causality

Consider a trivariate process $W_t = (X_t\,, Y_t\,, Z_t)'$ with the VAR(6) representation:

$$
\begin{aligned}
W_t \;&= \begin{bmatrix} .30 & .05 & .40 \\ -.30 & .30 & -.40 \\ -.50 & -.10 & .30 \end{bmatrix} W_{t-1} + \begin{bmatrix} -.30 & .01 & .40 \\ .30 & .10 & .10 \\ .30 & .10 & .10 \end{bmatrix} W_{t-2} \\
&+ \begin{bmatrix} -.20 & .03525 & -.10 \\ -.50 & .30 & -.50 \\ -.20 & .20 & -.10 \end{bmatrix} W_{t-3} + \begin{bmatrix} .20 & -.1256 & -.10 \\ .30 & -.10 & -.10 \\ .10 & 0 & -.10 \end{bmatrix} W_{t-4} \\
&+ \begin{bmatrix} -.10 & -.12659625 & .50 \\ -.10 & -.10 & 0 \\ .10 & 0 & -.10 \end{bmatrix} W_{t-5} + \begin{bmatrix} .10 & .02820225 & .40 \\ .30 & .30 & .40 \\ .30 & -.20 & .30 \end{bmatrix} W_{t-6} + u_t\,.
\end{aligned}
\tag{6.11}
$$

In the above model, the coefficients $\pi_{XYj}\,,\ j = 2,\ \ldots, 6,$ were chosen so that $\pi_{XY1}^{(h)} = 0$, $h = 2, \ldots, 6$ (the latter are reported with a higher precision to make the results easily checkable). The following table gives the coefficients $\pi_{XYj}^{(h)}$ , for $j = 1, 2, 3, 4, 5, 6,$ and $h = 1, 2, 3, 4, 5, 6.$

| **Table** 6.11**:** $\pi^{(h)}_{XYj} \times 10^3$ **corresponding to (**6.11**)** | | | | | | |
|---|---|---|---|---|---|---|
| h \ j | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 50.0 | 10.0 | 35.3 | -125.6 | -126.6 | 28.2 |
| 2 | 0 | 83.3 | -20.0 | -169.3 | -14.8 | -56.5 |
| 3 | 0 | 37.7 | -60.3 | 38.6 | -2.7 | -136.0 |
| 4 | 0 | -58.1 | 33.9 | 71.7 | -61.0 | -32.9 |
| 5 | 0 | -16.3 | -26.9 | -77.3 | -49.4 | 106.8 |
| 6 | 0 | -14.7 | -45.8 | -109.1 | 46.7 | -1.4 |

## 7. Causality at different horizons in macroeconomic and financial data

Empirical studies of causality at different horizons

1. Dufour, Pelletier, and Renault (2006): tests of causality at different horizons on monetary policy data previously studied by Bernanke and Mihov (1998).

2. Dufiour and Tessier (2006): tests of causality at different horizons to study the relationship between monetary policy variables and stock prices.

3. Dufour and Taamouti (2008): causality measures at different horizons on monetary policy data previously studied by Bernanke and Mihov (1998).

4. Dufour, Garcia, and Taamouti (2008): causality measures to study the interactions between stock returns, realized volatility and implied volatility, based on high-frequency data on S&P 500 Index futures contracts.

   This sheds light on the relative merits of the leverage hypothesis and the volatility feedback.

In Dufour, Pelletier, and Renault (2006), we reconsider the data set used by Bernanke and Mihov (1998) in order to study United States monetary policy: monthly observations (January 1965 to December 1996, 384 observations) on:

1. nonborrowed reserves ($NBR$, also denoted $w_1$);
2. the federal funds rate $(r,\ w_2)$;
3. the GDP deflator $(P,\ w_3)$;
4. real GDP $(GDP,\ w_4)$.

We propose a simple to implement linear estimation method in conjunction with bootstrapping to test causality at different horizons.

## 8. Conclusion

1. Econometric methods can provide useful information in assessing structural models and issues related to causal links.

2. It is important to formulate clearly what we mean we speak of simultaneity problems and causal links.

3. It is important to be careful with respect to finite-sample issues. Nowadays simulation-based procedures provide efficient solutions, or at least reasonable quick fixes to many difficult econometric problems.

Table 1. Summary of causality relations at various horizons for series in first difference

| h | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $NBR \nrightarrow r$ | ⋆⋆ | | | | | | | | | | | |
| $r \nrightarrow NBR$ | | | | | | | | | | | | |
| $NBR \nrightarrow P$ | ⋆⋆ | ⋆⋆ | ⋆⋆ | | | | ⋆ | | | | | |
| $P \nrightarrow NBR$ | | | | | | | | | | | | |
| $NBR \nrightarrow GDP$ | | | | | | | | | | | | |
| $GDP \nrightarrow NBR$ | | | | | | | | | | | | ⋆ |
| $r \nrightarrow P$ | | | | | | | | | | | | |
| $P \nrightarrow r$ | | | | | | | | | | | | |
| $r \nrightarrow GDP$ | | | ⋆ | ⋆ | ⋆ | ⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ |
| $GDP \nrightarrow r$ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | | | | | | | |
| $P \nrightarrow GDP$ | | | | | | | | | | | | |
| $GDP \nrightarrow P$ | | | | | | ⋆ | | ⋆ | ⋆ | | | |

| h | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $NBR \nrightarrow r$ | | | | | | | | | | | | |
| $r \nrightarrow NBR$ | | | | | | | | | | | | |
| $NBR \nrightarrow P$ | | | | | | | | | | | | |
| $P \nrightarrow NBR$ | | | | | | | | | | | | |
| $NBR \nrightarrow GDP$ | | | | | | | | | | | | |
| $GDP \nrightarrow NBR$ | | | | ⋆ | | | | | | | | |
| $r \nrightarrow P$ | | | | | | | | | | | | |
| $P \nrightarrow r$ | | | | | | | | | | | | |
| $r \nrightarrow GDP$ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆⋆ | ⋆ | ⋆ | | | | |
| $GDP \nrightarrow r$ | | | | | | | | | | | | |
| $P \nrightarrow GDP$ | | | | | | | | | | | | |
| $GDP \nrightarrow P$ | | | | | | | | | | | | |

Note _ The symbols ⋆ and ⋆⋆ indicate rejection of the non-causality hypothesis at the 10% and 5% levels respectively.

**References**


ANGRIST, J. D., AND A. B. KRUEGER (1991): "Does Compulsory School Attendance Affect Schooling and Earning?," *Quarterly Journal of Economics*, CVI, 979–1014.

BEAULIEU, M.-C., J.-M. DUFOUR, AND L. KHALAF (2005): "Testing Black's CAPM with Possibly Non-Gaussian Errors: An Exact Identification-Robust Simulation-Based Approach," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal.

——— (2006): "Testing Black's CAPM with Possibly Non-Gaussian Errors: An Exact Identification-Robust Simulation-Based Approach," Discussion paper, CIREQ, Université de Montréal and Université Laval.

——— (2007): "Finite-Sample Multivariate Tests of Asset Pricing Models with Coskewness," *Com-*

*putational Statistics and Data Analysis*, forthcoming.

BERNANKE, B. S., AND I. MIHOV (1998): "Measuring Monetary Policy," *The Quarterly Journal of Economics*, 113(3), 869–902.

BOUDJELLABA, H., J.-M. DUFOUR, AND R. ROY (1992): "Testing Causality Between Two Vectors in Multivariate ARMA Models," *Journal of the American Statistical Association*, 87(420), 1082–1090.

——— (1994): "Simplified Conditions for Non-Causality between Two Vectors in Multivariate ARMA Models," *Journal of Econometrics*, 63, 271–287.

BOUND, J., D. A. JAEGER, AND R. BAKER (1993): "The Cure can be Worse than the Disease: A Cautionary Tale Regarding Instrumental Variables," Technical Working Paper 137, National Bureau of Economic Research, Cambridge, MA.

BOUND, J., D. A. JAEGER, AND R. M. BAKER (1995): "Problems With Instrumental Variables

Estimation When the Correlation Between the Instruments and the Endogenous Explanatory Variable Is Weak," *Journal of the American Statistical Association*, 90, 443–450.

BUSE, A. (1992): "The Bias of Instrumental Variables Estimators," *Econometrica*, 60, 173–180.

CHOI, I., AND P. C. B. PHILLIPS (1992): "Asymptotic and Finite Sample Distribution Theory for IV Estimators and Tests in Partially Identified Structural Equations," *Journal of Econometrics*, 51, 113–150.

DOKO TCHATOKA, F., AND J.-M. DUFOUR (2008): "Instrument Endogeneity and Identification-Robust Tests: Some Analytical Results," *Journal of Statistical Planning and Inference*, 138, 2649–2661.

DUFIOUR, J.-M., AND D. TESSIER (2006): "Short-Run and Long-Run Causality Between Monetary Policy Variables and Stock Prices," Discussion Paper 2006-39, Bank of Canada, Ottawa, Canada.

DUFOUR, J.-M. (1997): "Some Impossibility Theorems in Econometrics, with Applications to Structural and Dynamic Models," *Econometrica*, 65, 1365–1389.

——— (2003): "Identification, Weak Instruments and Statistical Inference in Econometrics," *Canadian Journal of Economics*, 36(4), 767–808.

DUFOUR, J.-M., R. GARCIA, AND A. TAAMOUTI (2008): "Measuring Causality Between Volatility and Returns with High-Frequency Data," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal, Montréal, Canada.

DUFOUR, J.-M., AND J. JASIAK (1993): "Finite Sample Inference Methods for Simultaneous Equations and Models with Unobserved and Generated Regressors," Discussion paper, C.R.D.E., Université de Montréal, 38 pages.

DUFOUR, J.-M., AND J. JASIAK (2001): "Finite Sample Limited Information Inference Methods for Structural Equations and Models with Generated

Regressors," *International Economic Review*, 42, 815–843.

DUFOUR, J.-M., AND T. JOUINI (2006): "Finite-Sample Simulation-Based Tests in VAR Models with Applications to Granger Causality Testing," *Journal of Econometrics*, 135(1-2), 229–254.

DUFOUR, J.-M., L. KHALAF, AND M. KICHIAN (2006a): "Inflation Dynamics and the New Keynesian Phillips Curve: An Identification Robust Econometric Analysis," *Journal of Economic Dynamics and Control*, 30(9-10), 1707–1727.

——— (2006b): "Structural Estimation and Evaluation of Calvo-Style Inflation Models," Discussion paper, Centre de recherche et développement en économique (CRDE), Université de Montréal, and Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Montréal, Canada.

——— (2006c): "Structural Multi-Equation Macroeconomic Models: A System-Based Estimation and Evaluation Approach," Discussion paper, McGill University (Department of Economics), Montréal, and Bank of Canada, Ottawa, Canada, Montréal, Canada.

——— (2006d): "Structural Multi-Equation Macroeconomic Models: A System-Based Estimation and Evaluation Approach," Discussion paper, Centre de recherche et développement en économique (CRDE), Université de Montréal, and Centre interuniversitaire de recherche en analyse des organisations (CIRANO), Montréal, Canada.

——— (2007): "Which NKPC? Identification-Robust Estimates of Price Stickiness and Real Wage Rigidity," Discussion paper, McGill University (Department of Economics), Montréal, and Bank of Canada, Ottawa, Canada.

——— (2008): "How Much Do Real Wage Rigidities Matter for Canadian Inflation?," Discussion paper, McGill University (Department of Economics), Montréal, and Bank of Canada, Ottawa, Canada, Montréal, Canada.

DUFOUR, J.-M., S. NSIRI, AND D. TESSIER (1994): "Parsimonious Autoregressive Conditions for Non-Causality in Multivariate ARMA Models," in *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, pp. 129–134, Washington (D.C.).

DUFOUR, J.-M., AND D. PELLETIER (2005): "Practical Methods for Modelling Weak VARMA Processes: Identification, Estimation and Specification with a Macroeconomic Application," Discussion paper, Département de sciences économiques and CIREQ, Université de Montréal, Montréal, Canada.

DUFOUR, J.-M., D. PELLETIER, AND É. RENAULT (2006): "Short Run and Long Run Causality in Time Series: Inference," *Journal of Econometrics*, 132(2), 337–362.

DUFOUR, J.-M., AND E. RENAULT (1998): "Short-Run and Long-Run Causality in Time Series: Theory," *Econometrica*, 66, 1099–1125.

DUFOUR, J.-M., AND A. TAAMOUTI (2008): "Short and Long Run Causality Measures: Theory and Inference," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal, Montréal, Canada.

DUFOUR, J.-M., AND M. TAAMOUTI (2001): “Point-Optimal Instruments and Generalized Anderson-Rubin Procedures for Nonlinear Models,” Discussion paper, C.R.D.E., Université de Montréal.

——— (2003a): “On Methods for Selecting Instruments,” Discussion paper, C.R.D.E., Université de Montréal.

——— (2003b): “Point-Optimal Instruments and Generalized Anderson-Rubin Procedures for Nonlinear Models,” Discussion paper, C.R.D.E., Université de Montréal.

——— (2005): “Projection-Based Statistical Inference in Linear Structural Models with Possibly Weak Instruments,” *Econometrica*, 73(4), 1351–1365.

——— (2006): “Further Results on Projection-Based Inference in IV Regressions with Weak, Collinear or Missing Instruments,” *Journal of Econometrics*, forthcoming.

——— (2007): "Further Results on Projection-Based Inference in IV Regressions with Weak, Collinear or Missing Instruments," *Journal of Econometrics*, 139(1), 133–153.

DUFOUR, J.-M., AND D. TESSIER (1993): "On the Relationship Between Impulse Response Analysis, Innovation Accounting and Granger Causality," *Economics Letters*, 42, 327–333.

——— (1997a): "La causalité entre la monnaie et le revenu: une analyse de causalité basée sur un modèle ARMA-échelon," *L'Actualité économique*, 73, 351–366.

——— (1997b): "La causalité entre la monnaie et le revenu: une analyse de causalité basée sur un modèle ARMA-échelon," in *Économétrie appliquée*, ed. by C. Gouriéroux, and C. Montmarquette. Economica, Paris.

DUFOUR, J.-M., AND P. VALÉRY (2005): "Exact and Asymptotic Tests for Possibly Non-Regular Hypotheses on Stochastic Volatility Models," Discussion paper, Centre interuniversitaire de recherche en analyse des organisations (CIRANO) and Centre interuniversitaire de recherche en économie quantitative (CIREQ), Université de Montréal.

GLESER, L. J., AND J. T. HWANG (1987): "The Nonexistence of $100(1-\alpha)$ Confidence Sets of Finite Expected Diameter in Errors in Variables and Related Models," *The Annals of Statistics*, 15, 1351–1362.

GRANGER, C. W. J. (1969): "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," *Econometrica*, 37, 424–459.

HALL, A. R., G. D. RUDEBUSCH, AND D. W. WILCOX (1996): "Judging Instrument Relevance in Instrumental Variables Estimation," *International Economic Review*, 37, 283–298.

HILLIER, G. H. (1990): "On the Normalization of Structural Equations: Properties of Direction Estimators," *Econometrica*, 58, 1181–1194.

KLEIBERGEN, F. (2001): "Testing Subsets of Structural Coefficients in the IV Regression Model," Discussion paper, Department of Quantitative Economics, University of Amsterdam.

——— (2002): "Pivotal Statistics for Testing Structural Parameters in Instrumental Variables Regression," *Econometrica*, 70(5), 1781–1803.

KOSCHAT, M. A. (1987): "A Characterization of the Fieller Solution," *The Annals of Statistics*, 15, 462–468.

LÜTKEPOHL, H. (1993): "Testing for Causation Between Two Variables in Higher Dimensional VAR Models," in *Studies in Applied Econometrics*, ed. by H. Schneeweiss, and K. Zimmermann. Springer-Verlag, Heidelberg.

MA, A. (2002): "GMM Estimation of the New Phillips Curve," *Economic Letters*, 76, 411–417.

MADDALA, G. S., AND J. JEONG (1992): "On the Exact Small Sample Distribution of the Instrumental Variable Estimator," *Econometrica*, 60, 181–183.

MCMANUS, D. A., J. C. NANKERVIS, AND N. E. SAVIN (1994): "Multiple Optima and Asymptotic Approximations in the Partial Adjustment Model," *Journal of Econometrics*, 62, 91–128.

MOREIRA, M. J. (2001): "Tests With Correct Size

When Instruments Can Be Arbitrarily Weak," Discussion paper, Department of Economics, Harvard University, Cambridge, Massachusetts.

——— (2002): "A Conditional Likelihood Ratio Test for Structural Models," Discussion paper, Department of Economics, Harvard University, Cambridge, Massachusetts.

NASON, J. M., AND G. W. SMITH (2005): "Identifying the New Keynesian Phillips Curve," Discussion Paper 2005-1, Federal Reserve Bank of Atlanta.

NELSON, C. R., AND R. STARTZ (1990a): "The Distribution of the Instrumental Variable Estimator and its $t$-ratio When the Instrument is a Poor One," *Journal of Business*, 63, 125–140.

——— (1990b): "Some Further Results on the Exact Small Properties of the Instrumental Variable Estimator," *Econometrica*, 58, 967–976.

PERRON, B. (1999): "Semi-Parametric Weak Instrument Regressions with an Application to the

Risk Return Trade-Off," Discussion Paper 0199, C.R.D.E., Université de Montréal.

PHILLIPS, P. C. B. (1984): "The Exact Distribution of LIML: I," *International Economic Review*, 25, 249–261.

——— (1985): "The Exact Distribution of LIML: II," *International Economic Review*, 26, 21–36.

——— (1989): "Partially Identified Econometric Models," *Econometric Theory*, 5, 181–240.

SARGAN, J. D. (1983): "Identification and Lack of Identification," *Econometrica*, 51, 1605–1633.

SHEA, J. (1997): "Instrument Relevance in Multivariate Linear Models: A Simple Measure," *Review of Economics and Statistics*, LXXIX, 348–352.

STAIGER, D., AND J. H. STOCK (1997): "Instrumental Variables Regression with Weak Instruments," *Econometrica*, 65(3), 557–586.

STARTZ, R., C. R. NELSON, AND E. ZIVOT (1999): "Improved Inference for the Instrumental Variable

Estimator," Discussion paper, Department of Economics, University of Washington.

STOCK, J. H., AND J. H. WRIGHT (2000): "GMM with Weak Identification," *Econometrica*, 68, 1097–1126.

STOCK, J. H., J. H. WRIGHT, AND M. YOGO (2002): "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments," *Journal of Business and Economic Statistics*, 20(4), 518–529.

STOCK, J. H., AND M. YOGO (2002): "Testing for Weak Instruments in Linear IV Regression," Discussion Paper 284, N.B.E.R., Harvard University, Cambridge, Massachusetts.

WANG, J., AND E. ZIVOT (1998): "Inference on Structural Parameters in Instrumental Variables Regression with Weak Instruments," *Econometrica*, 66(6), 1389–1404.

WIENER, N. (1956): "The Theory of Prediction," in *The Theory of Prediction*, ed. by E. F. Beckenback, chap. 8. McGraw-Hill, New York.

Zivot, E., R. Startz, and C. R. Nelson (1998): "Valid Confidence Intervals and Inference in the Presence of Weak Instruments," *International Economic Review*, 39, 1119–1144.